Science Olympiad
Machines C
BEARSO Invitational

October 10, 2020

Directions:

- Do **not** open the test until told to start!
- Each team will be given **50 minutes** to complete the test.
- There are two sections: **Section A** (Multiple Choice) and **Section B** (Free Response).
- Write **all answers on the lines on the answer pages**. Any marks elsewhere will not be scored. And make sure to include your **team name** and **team number** at the top of all answer pages.
- Feel free to take apart the test, so long as it is restapled before it is turned in.
- Do not worry about significant figures. Just make sure to use 3 or more in your answers.
- Whenever needed, take the acceleration of gravity, g, to be $9.81\,\mathrm{m\,s^{-2}}$.
- Ties will be broken by comparing points earned on the following sections/questions, in order: Section B, §B1, §B2, . . . , §B7, §B1ai, §B1aii, . . . , §B7biii, §A40, §A39, . . . , §A1.
- Best of luck! And may the odds be ever in your favor.

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Feedback? **Test Code**: *2021BEARSO-MachinesC-Shell*

Section A: Multiple Choice

Each question in this section is worth two points, for a total of 80 points.

Questions 1-7 refer to Yovani, a construction worker digging a pit.

1. To break into the dirt, Yovani points the shovel downwards and pushes on the shovel head. What simple machine best describes the shovel head?
   A. Screw
   B. Inclined Plane
   C. Lever
   D. Wedge

2. The shovel head is placed at a 80° angle with respect to the ground. If Yovani pushes directly down with a force of 300 N, what is the actual effort force on the shovel head?
   A. 300 N
   B. 295 N
   C. 275 N
   D. 267 N

3. Yovani holds the shovel by placing his left hand a third of the way up the shovel and his right hand at the top of the shovel's handle. He applies a downwards force with his right hand. What is the lever class of the shovel and where is its fulcrum?
   A. Class 1; at his right hand
   B. Class 3; at his right hand
   C. Class 1; at his left hand
   D. Class 3; at his left hand

4. Yovani exerts a downwards force of 100 N with his right hand to lift 150 N of dirt. What is the efficiency of the shovel?
   A. 50%
   B. 75%
   C. 80%
   D. 90%

5. What is the mass of the shovel head? Assume the mass of the shovel head is much greater than the mass of the handle.
   A. 1 kg
   B. 2 kg
   C. 5 kg
   D. 15 kg

6. Yovani shovels 1000 N of dirt into a bucket to be hoisted out of the pit. However, Yovani can only pull with a force of 200 N, what must the AMA of the hoist pulley system be?
   A. 2
   B. 3
   C. 4
   D. 5

7. The dirt is 5 m deep. How much power must Yovani exert to lift the 1000 N of dirt in 2 s?
   A. 500 W
   B. 1000 W
   C. 1500 W
   D. 2500 W

Questions 8-12 refer to Summer, a student working part-time over the summer as a mover. She loads boxes into the back of a truck by pushing boxes up a ramp. The ramp is 11 m long and makes a 20° angle with the ground.

8. What is the IMA of the ramp?
   A. 2.9
   B. 2.5
   C. 1.6
   D. 1.1

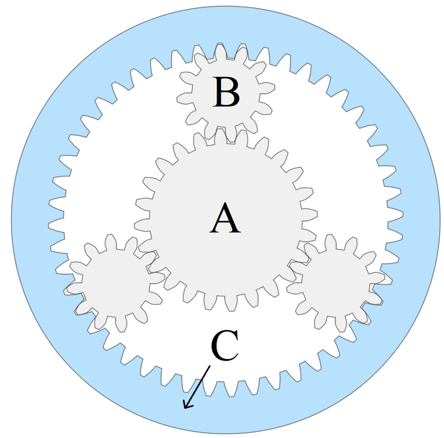
9. Summer pushes a 100 N box halfway up the ramp. What is its change in potential energy?
   A. 376 J
   B. 188 J
   C. −376 J
   D. −188 J

10. If Summer pushes with a horizontal force of 150 N to move a 200 N box with a constant velocity, what is the coefficient of kinetic friction between the ramp and the box?
   A. 0.1
   B. 0.2
   C. 0.3
   D. 0.4

11. Summer pushes a 300 N box two thirds of the way up the ramp, decides to take her lunch break, and leaves the box on the ramp. What is the normal force on the ramp from the ground?
   A. 94 N
   B. 100 N
   C. 115 N
   D. 150 N

12. Summer finishes her lunch break and helps move the last item: a grand piano. It weighs a staggering 360 kg. Even though it's on wheels, it still takes 5 people each pushing along the direction of the ramp with a force of 300 N to move it. If they all decide to take a break while pushing it, will the piano stay in place or roll back down the ramp?
   A. It will roll down, because $\eta < 50\%$.
   B. It will roll down, because $\eta \geq 75\%$.
   C. It will stay in place, because $\eta \geq 50\%$.
   D. It will stay in place, because $\eta < 75\%$.

Questions 12-17 refer to the gear hub depicted below. Gears A, B, and C have 24, 12, and 48 teeth, respectively.



13. What type of gear system is gear A and B?
   A. Planetary gear
   B. Coupled gear
   C. Sun and planet gear
   D. Cycloidal gear

14. What type of gear is gear C?
   A. Bevel gear
   B. Face gear
   C. Internal gear
   D. Spur gear

15. All three gears are fixed but can freely rotate. Gear C rotates counterclockwise with 16 rpm. In what direction do gears A and B rotate in?
   A. A: counterclockwise; B: counterclockwise
   B. A: counterclockwise; B: clockwise
   C. A: clockwise; B: counterclockwise
   D. A: clockwise; B: clockwise

16. Gear A is fixed and cannot rotate. Gear C rotates clockwise with 8 rpm. What is the angular speed of gear B?
   A. 64 rpm
   B. 32 rpm
   C. 24 rpm
   D. 16 rpm

17. Gear C is fixed and cannot rotate. Gear A rotates counterclockwise with 6 rpm. How much time does it take gear B to make one full revolution around gear A?

    A. 60 s

    B. 30 s

    C. 20 s

    D. 10 s

18. Gears hubs like this are used in bicycles. Since cyclists want to be as efficient as possible, these hubs are designed for efficiencies up to 95-98%. If a cyclist rides for 1 hour with an average speed of 4 mph and an average pedaling force of 140 lbf, how much energy could be lost?

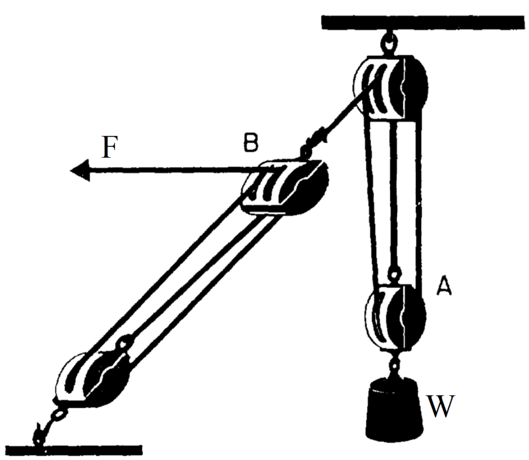
    A. 30 kJ

    B. 70 kJ

    C. 150 kJ

    D. 210 kJ

Questions 19-23 refer to Bobby, a sailor on the SS Archy. He is being trained on the pulley systems aboard the ship. The double pulley system referenced is depicted below with tackles A and B labelled.



19. What type of tackle are tackles A and B?

    A. A: luff tackle; B: gun tackle

    B. A: luff tackle; B: luff tackle

    C. A: double tackle; B: double tackle

    D. A: double tackle; B: gyn tackle

20. What is the IMA of tackle A?

    A. 2

    B. 3

    C. 4

    D. 5

21. Bobby pulls with a force F of 150 N to move a 1200 N weight. What is the efficiency of the pulley system?

    A. 89%

    B. 80%

    C. 75%

    D. 67%

22. The final pulley Bobby learns about is a differential pulley with a large pulley of radius 50 cm and a small pulley of radius 40 cm. How much force must he exert to lift the same 1200 N weight?

    A. 120 N

    B. 160 N

    C. 200 N

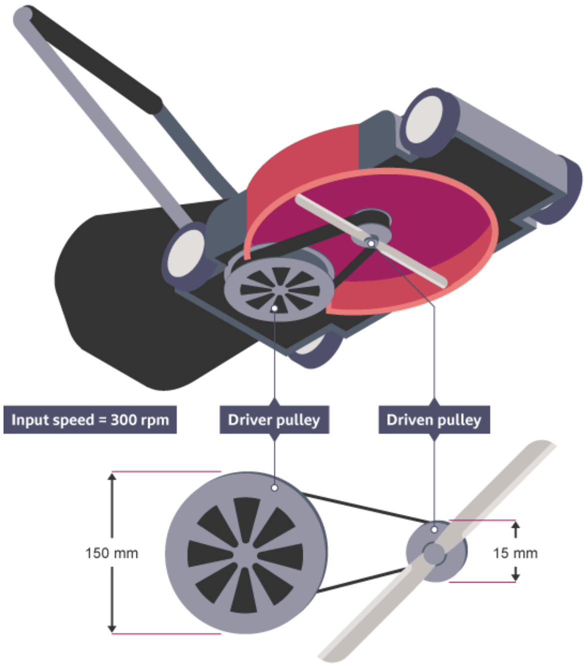
    D. 300 N

23. If that differential pulley has an efficiency of 78%, should Bobby use the double pulley system instead?

    A. Yes

    B. No

Questions 24-27 refer to John's lawn mower, depicted below.



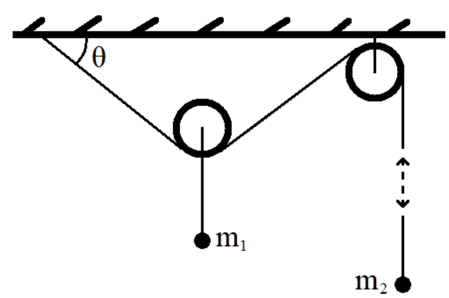
24. What is the rotational speed of the driven pulley?
   A. 3000 rpm
   B. 4000 rpm
   C. 6000 rpm
   D. 8000 rpm

25. John notices the lawn mower's blades are spinning slower than when he first bought it. Which of the following is a possible reason for this inefficiency?
   A. The increased tension on the belt snapped it.
   B. The high rpm on the pulleys strained them.
   C. The continued use of the machine smoothed the belt surface.
   D. The large amount of grass cut by the machine dulled the blades.

26. If the centers of the two pulleys are 17 cm apart, how long must the belt be? Assume its thickness is negligible.
   A. 626 mm
   B. 687 mm
   C. 714 mm
   D. 729 mm

27. How much longer must the belt be if John wants the driven pulley to rotate in the opposite direction?
   A. 264 mm
   B. 307 mm
   C. 322 mm
   D. 365 mm

Questions 28-30 refer to June, a Science Olympiad competitor. June misread the Machines rules and instead of a lever device, she built a pulley device, shown in the diagram below. Assume the masses are point masses, the pulleys are ideal, and the rope is long enough so m_2 never goes over the pulley.



28. June is given 3 masses (m_A, m_B, and m_C) of unknown weight. She places mass A where m_1 is and mass B where m_2 is. If $m_A \geq 2m_B$, how would you describe the motion of the device after a long time?
   A. m_A is accelerating downwards.
   B. m_A is moving downwards with a constant velocity.
   C. m_B is accelerating downwards.
   D. m_B is moving downwards with a constant velocity.
   E. It reaches an equilibrium point.

29. June now places mass C where m_1 is and mass A where m_2 is. After a long time, θ equals 60°. What is the mass ratio m_A/m_C?
    A. 1.73
    B. 1.15
    C. 0.87
    D. 0.58

30. June hopes to do well at her next invitational. Which of the following would **not** improve the precision of her device?
    A. Replacing the pulleys with lighter ones.
    B. Moving the right pulley down.
    C. Increasing the length of the rope.
    D. Buying a stronger rope.

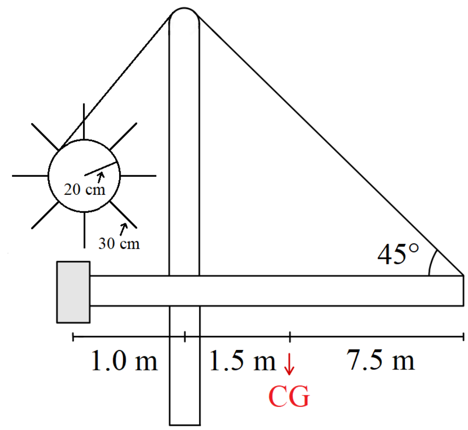
Questions 31-33 refer to James the mechanic and his screw jack. The screw jack has a handle length of 70 cm and is self-locking. The screw inside is double started with a pitch of 3 mm.

31. How many revolutions will it take to lift a 2100 kg car by 18 cm?
    A. 90
    B. 60
    C. 45
    D. 30

32. What is the IMA of the screw?
    A. 367
    B. 733
    C. 1100
    D. 1467

33. If James had to exert a force of 100 N to lift the car, what is the minimum torque to lower the load?
    A. 72 N m
    B. 50 N m
    C. 43 N m
    D. 30 N m

Questions 34-39 refer to the diagram of Yvette's drawbridge shown below. The drawbridge consists of a wooden bridge, a counterweight, a wooden post, a windlass with attached spokes, and a rope to lift the bridge. Assume the rope is ideal and all materials are uniform. The center of gravity of the bridge and counterweight system is marked as CG.



34. How would the removal of the counterweight affect the system?
    I. Increase the force on the lowered bridge.
    II. Shift the center of gravity leftwards.
    III. Decrease the mechanical advantage.
    A. I only
    B. III only
    C. I and III
    D. II and III
    E. I, II, and III

35. The wooden bridge and counterweight have a total mass of 1000 kg. What is the mass of the counterweight?
    A. 500 kg
    B. 400 kg
    C. 333 kg
    D. 250 kg

36. The wooden bridge and counterweight act as a lever beam. What is the lever class of this combination and what is it's IMA?
    A. Class 3; 6
    B. Class 3; 5
    C. Class 2; 6
    D. Class 2; 5

37. The drawbridge is raised up at a constant angular velocity. How does the tension in the rope change with time?

    A. It decreases then increases

    B. It increases then decreases

    C. It only decreases

    D. It only increases

    E. It does not change

38. The windlass is a wheel with spokes attached around it to help give additional leverage. How many times does Yvette have to rotate the windlass to raise the drawbridge by 60 degrees?

    A. 2 to 4 turns

    B. 4 to 6 turns

    C. 6 to 8 turns

    D. 8 to 10 turns

39. **This question was removed from scoring.**

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40. There are four reversible (the input and output can be switched) simple machines with IMAs of 2, 3, 5, and 7. How many possible unique IMA values can be made through the combinations of these four machines? Assume that using none of the machines results in an IMA of 1.

    A. 1 to 15

    B. 16 to 40

    C. 41 to 80

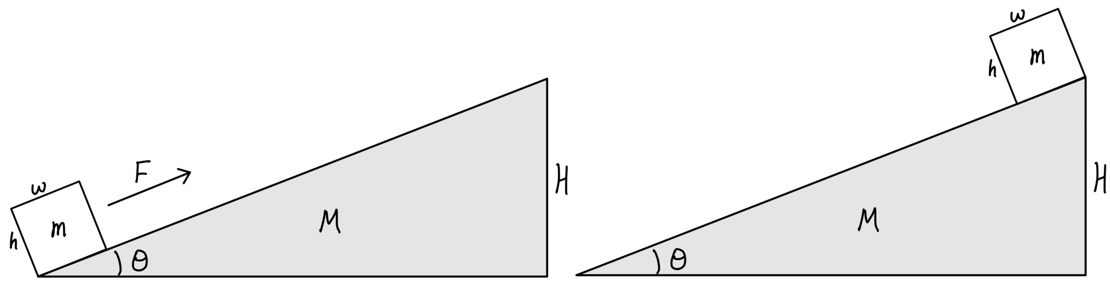
    D. 81 to 150

    E. ≥ 151

Section B: Free Response

Points are shown for each question or sub-question, for a total of 120 points.

1. (24 points) An inclined plane of angle $\theta = 25°$, height $H = 4\,\text{m}$, and mass $M = 20\,\text{kg}$ is locked to the floor. A block of mass $m = 3\,\text{kg}$, width $w = 50\,\text{cm}$, and height $h = 30\,\text{cm}$ lies on the end of the plane and is pushed up the plane. Both objects are of uniform density. Below is a diagram of the system initially (on the left) and the system after the block has been raised (on the right).



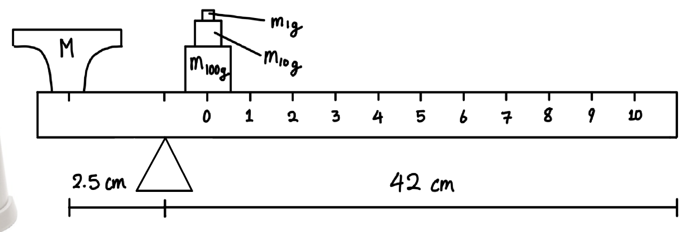
   (a) Assuming the inclined plane and block are frictionless, find:
      i. (2 points) The force required to keep the block at rest, in N.
      ii. (2 points) The energy required to lift the block to the top of the plane (right diagram), in J.

   (b) In reality, the system is not ideal. It takes 150 J of work to lift the block from the bottom of the plane (left diagram) to the top of the plane (right diagram).
      i. (3 points) Calculate the AMA of the ramp.
      ii. (2 points) One possible reason for this energy loss is air resistance. Name another likely reason for this energy loss.
      iii. (3 points) If the primary energy loss occurred through air resistance, describe how it can be minimized.

   (c) Now let's return to the original assumption of ideal conditions: the block, plane, and floor are all frictionless. Once the block is lifted to the top of the plane (right diagram), both the block and inclined plane are simultaneously released, so that both can slide freely on each other and on the floor.
      i. (3 points) Qualitatively describe what happens to the system.
      ii. (4 points) Calculate the acceleration of the inclined plane right after the release, in $\text{m}\,\text{s}^{-2}$.
      iii. (5 points) Find the velocity of the block a long time after the release, in $\text{m}\,\text{s}^{-1}$.

2. (12 points) A triple beam balance is an instrument used to measure mass very precisely. Shown below is a picture of it (left) and a lever representation of it (right). There are 3 riders ($m_{100\,\mathrm{g}}$, $m_{10\,\mathrm{g}}$, and $m_{1\,\mathrm{g}}$), a weighing pan of mass M, and a lever with linear density $\lambda = 1\,\mathrm{g\,cm^{-1}}$. All gaps to the right of the fulcrum are of equal length.

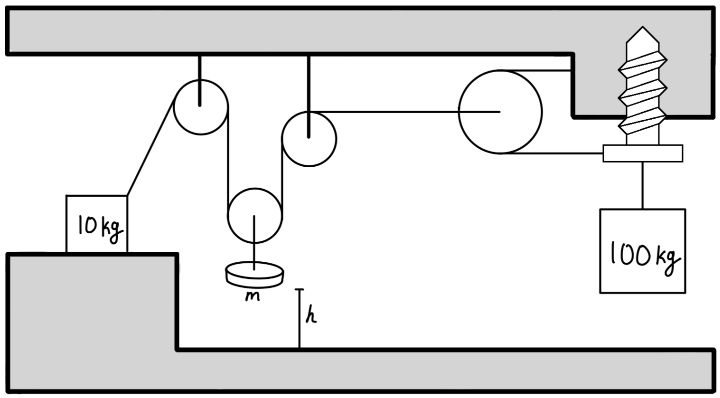
   To weigh an object, it is placed on the weighing pan. Then, the riders are adjusted such that the balance is in equilibrium. The final locations of the riders depends on the mass of the object. For example, if a 461 g object is weighed, the $m_{100\,\mathrm{g}}$, $m_{10\,\mathrm{g}}$, and $m_{1\,\mathrm{g}}$ riders will be placed over the ticks marked 4, 6, and 1, respectively, so that the balance is in equilibrium. Assume the center of mass of the weighing pan and the weighed objects are above the left tick mark.



   (a) Find, in grams:
      i. (3 points) $m_{100\,\mathrm{g}} + m_{10\,\mathrm{g}} + m_{1\,\mathrm{g}}$. *(Hint: The mass of m_{100g}, m_{10g}, and m_{1g} are not 100 g, 10 g, and 1 g. What does it means to move the masses by one tick mark?)*
      ii. (4 points) M.

   (b) There is a frictional moment of $2.5 \times 10^{-6}\,\mathrm{N\,m}$ at the fulcrum.
      i. (3 points) What is the reading error of the balance, in grams?
      ii. (2 points) How could the balance be modified to lower its error?

3. (12 points) A compound machine is shown below. It consists of a frictionless and massless single-threaded screw, an ideal pulley system with two fixed and two movable pulleys connected with lightweight, flexible cords, a 10 kg block on a rough surface, a 100 kg block attached to the bottom of the screw, and a disk of mass m lifted a distance of $h = 2\,\mathrm{m}$ off the ground. The screw has a 7 mm pitch, a 3 cm shaft radius, and a 5 cm cap radius where a cord is attached to. The aforementioned cord is attached to the edge of the cap radius such that pulling on it tightens the screw. The machine is currently in equilibrium.



(a) (2 points) What is the mass of the disk, in kg? *(Hint: Treat the block as an immovable surface.)*

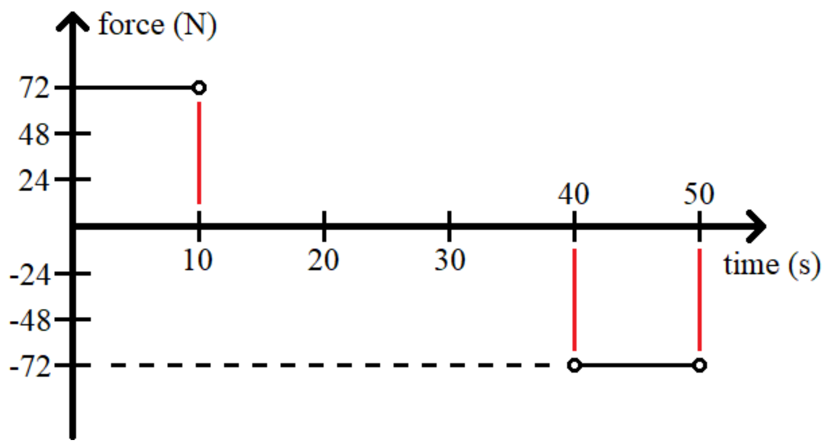
(b) (3 points) If the cord is connected to the 10 kg block at a 55° angle with respect to the ground, what are the possible values for the coefficient of static friction between the block and the floor?

(c) (4 points) The cord connected to the 100 kg block is cut. What is the upwards velocity of the screw right when the disk has fallen half the distance to the ground, in $\mathrm{cm\,s^{-1}}$?

(d) (3 points) How far upwards would the screw have moved from its initial position once the disk hits the ground, in cm?

4. (21 points) An elevator is designed with a counterweight system, connected together with a lightweight wheel and axle, such that the elevator cab and counterweight are in equilibrium when empty. The elevator cab weighs 1200 kg and is designed to hold 1000 kg. The wheel has a radius of 1.75 m and the axle has a diameter of 2.5 m. All of these components are connected with long lengths of lightweight cables.

   (a) (2 points) If the mass of the counterweight should be less than the mass of the elevator cab, what should the mass of the counterweight be, in kg, and what should the cables connect?



   (b) To test the elevator, the technician sends the elevator up four floors. Shown above is a diagram of the force applied to the elevator cab as a function of time. Positive force acts upwards on the cab.

      i. (3 points) Qualitatively describe the motion of the elevator cab.

      ii. (4 points) What is the speed of the counterweight after 15 s, in $\mathrm{m\,s^{-1}}$?

      iii. (3 points) What is the height of a floor, in meters?

   (c) The elevator is then boarded by three people, each weighing 80 kg, who all want to go up 9 m in 50 s. In order to give the passengers a smoother ride, the function of the force applied to the elevator cab is changed to be

$$F(t) = \alpha \sin(\beta t) + \gamma$$

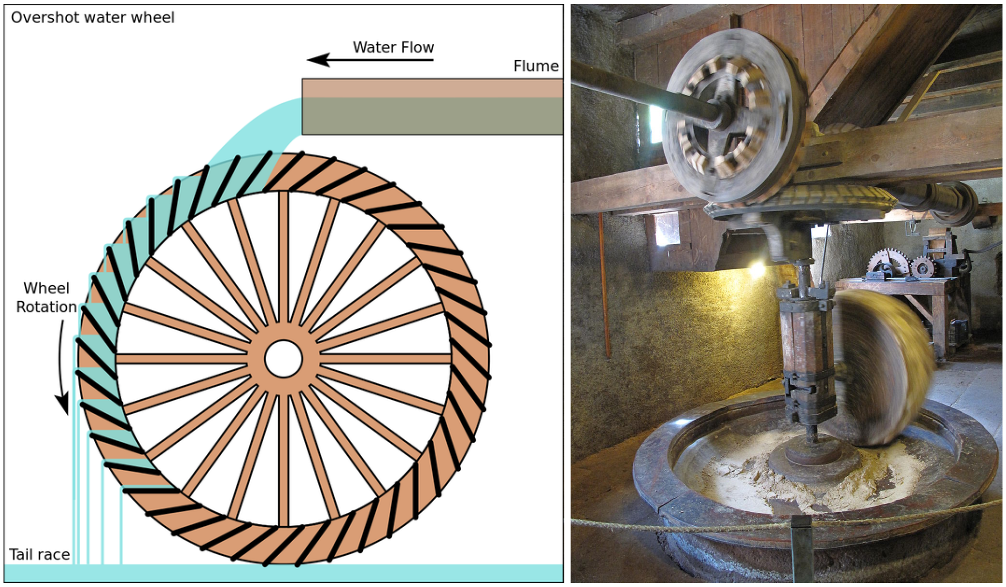
   where α, β, and γ are constants such that the elevator will only accelerate and decelerate once and that $F_{net}(0) = F_{net}(t_f) = 0$, where $F_{net}(t)$ is the net force on the elevator at time t and t_f is the time at which the elevator reaches a height of 9 m. Find, in their respective SI units:

      i. (4 points) α.

      ii. (3 points) β.

      iii. (2 points) γ.

5. (21 points) A watermill is a mill that uses hydropower.



(a) The diagram on the left shows the waterwheel that powers the mill. The water flows through a flume at a rate of $5.00 \times 10^4\ \mathrm{cm^3\,s^{-1}}$ and falls from a height 6.00 m above the water level. Recall the density of water is $1\ \mathrm{g\,mL^{-1}}$.

i. (5 points) Given the waterwheel rotates at 10 rpm, find the maximum torque on the central axle, in N m. Assume the diameter of the wheel is near the height of the flume, the water enters and exits at the top and bottom of the wheel, and the water is evenly distributed between the blades.

ii. (2 points) If the torque on the central axle is 2250 N m, what is the efficiency of the waterwheel?

iii. (2 points) Give two possible reasons for the waterwheel's inefficiency.

(b) The diagram on the right depicts the grinding mechanism of the mill. The primary axle rotates at 10 rpm with a torque of 2250 N m, the bevel gears on the primary and the secondary axle have a radius of 60 cm and 1 m, respectively, and the grindstone is a uniform disk of mass 500 kg and radius 1.5 m.

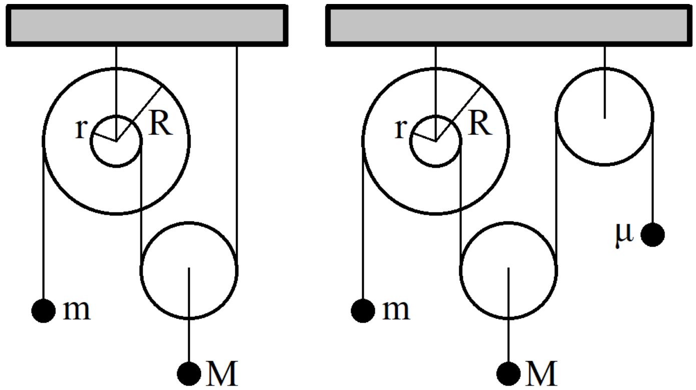
i. (2 points) The mill owner is considering changing the gears to have helical teeth. Give one advantage and one disadvantage of this change.

ii. (2 points) Calculate the IMA of the gears.

iii. (4 points) The coefficient of static and kinetic friction between the grindstone and the stone base is 0.8 and 0.5, respectively. What is the minimum distance it can be set from the secondary axle such that the grindstone rolls without slipping, in meters?

iv. (4 points) The grindstone is set 0.5 m from the secondary axle. What is the rate at which energy is being lost through friction, in W?

6. (12 points) Shown below on the left is a pulley system of two point masses (m and M) and three pulleys, one movable and two fixed and coaxially connected ($r = 25\,\text{cm}$ and $R = 90\,\text{cm}$). Assume ideal conditions.



   (a) (2 points) What is the IMA of the pulley system?
   (b) (3 points) If $m = 10\,\text{kg}$ and $M = 75\,\text{kg}$, Calculate the tension in the left and right strings, in N.

   The pulley system is modified so that another fixed pulley and a mass μ is added. The new system is shown above on the right.

   (c) (2 points) If $m = 50\,\text{kg}$, find M and μ such that the system is in equilibrium, in kg.
   (d) (5 points) Find the direction and magnitude of acceleration for m, in $\text{m}\,\text{s}^{-2}$, when $M = 30\,\text{kg}$ and $\mu = 90\,\text{kg}$.

7. (18 points) A 60 lbs wooden door is 80″ tall, 36″ wide, and of negligible thickness. It is designed to close on its own with a constant torque of 12 lbf·ft. Assume the door is currently opened by 30°. Below are the friction coefficients of common materials.

Material	μ_s	μ_k
Wood	0.40	0.25
Plastic	0.75	0.55
Rubber	0.95	0.80

   (a) A doorstop, of some angle θ, is placed at the end of the door. Assume the weight of the doorstop is negligible. Using the information provided above, find:
      i. (2 points) The magnitude and direction of the normal force on the door from doorstop, in terms of θ and lbf.
      ii. (3 points) The maximum possible angle of doorstops of each material that will still hold the door open, in degrees.
      iii. (2 points) The IMA of each of those doorstops found in the previous sub-question.
   (b) A rubber doorstop with an angle of 10° and a negligible weight to be placed at the end of the door and hold the door open. The door is then opened another 60° and released from rest.
      i. (5 points) Calculate the time it takes for the door to hit the doorstop, in seconds.
      ii. (4 points) Once the door hits the doorstop, there is a constant normal force of 30 lbf between the door and the doorstop. How much further must the door turn until it comes to rest, in degrees?
      iii. (2 points) How much work was done by friction, in N?