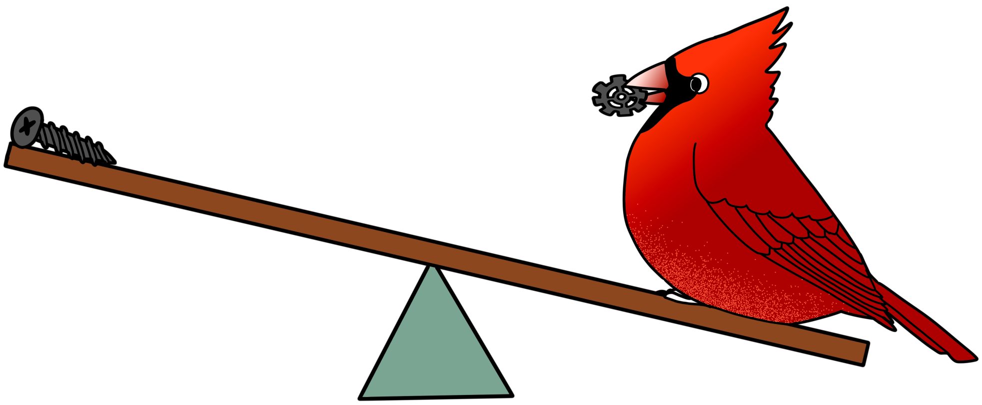
# Science Olympiad
# Machines C
# BirdSO Invitational

March 7-13, 2021



**Directions:**

- Each team will be given **50 minutes** to complete the test.
- There are two sections: **Section A** (Multiple Choice) and **Section B** (Free Response).
- Do not worry about significant figures. Just make sure to **use 3 or more in your answers**.
- Whenever needed, take the acceleration of gravity, $g$, to be $9.81\,\mathrm{m\,s^{-2}}$
- Tiebreakers, in order: §B, §B3, §B4, §A1, . . . , §A30.
- Best of luck! And may the odds be ever in your favor.


**Written by:**

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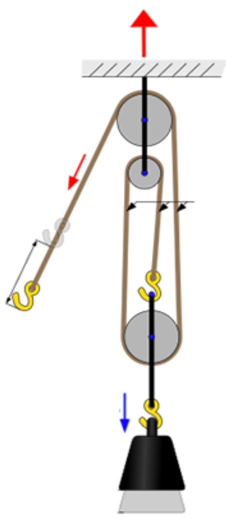
**Feedback?** **Test Code**: *2021BirdSO-MachinesC-Screw*

## Section A: Multiple Choice

**This section is comprised of three subsections, for a total of 60 points.**

1. What's the mechanical advantage of this pulley?



   A. 1
   B. 2
   C. 3
   D. 4
   E. 5

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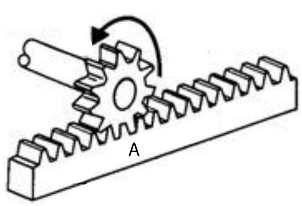
Questions 2-4 refer to the image below:




2. What type of gear is shown?
   A. Spur gear
   B. Helical gear
   C. Worm and worm gear
   D. Spur and stick
   E. Rack and pinion

3. What direction will A move if the gear is turning counterclockwise as shown in the image?
   A. Left
   B. Right
   C. None

4. Which of the following are true regarding this type of gear? (Select all that apply)
   A. It converts rotary motion into linear motion.
   B. It is designed so that rotational stability can improve the reliability of the gear system.
   C. This combination is often used as part of a linear actuator.
   D. The travel length is only limited by the length of structure A.
   E. A helical gear cannot be used as a driver gear for this gear system.

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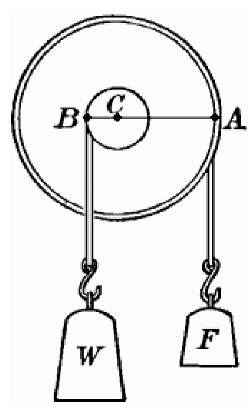
Questions 5-7 refer to Celeste's wheel and axle system, drawn below, which is defined by three points, $A$, $B$, and $C$. Let line segments $AB$ and $AC$ be 25 cm and 20 cm long, respectively. Also, let weight $W$ be 40 N.



5. What is the weight of $F$ if the system is ideal and in equilibrium?
   - A. 8 N
   - B. 10 N
   - C. 15 N
   - D. 24 N
   - E. 32 N

6. What is the actual mechanical advantage of the system if weight $F$ is 35 N and the system is in equilibrium?
   - A. 1.09
   - B. 1.14
   - C. 1.20
   - D. 1.43
   - E. 1.62

7. Let weight $F$ be 95 N and assume the system is ideal. What is the acceleration of weight $W$?
   - A. $214\,\mathrm{cm\,s^{-2}}$
   - B. $618\,\mathrm{cm\,s^{-2}}$
   - C. $794\,\mathrm{cm\,s^{-2}}$
   - D. $878\,\mathrm{cm\,s^{-2}}$
   - E. $8340\,\mathrm{cm\,s^{-2}}$

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8. Snowy the Polar Bear throws a fish-shaped rock vertically upward from the ground. The rock passes the top of a tower at $t = 2\,\mathrm{s}$ and later reaches its maximum height at $t = 3.14\,\mathrm{s}$. What is the height of the tower?
   - A. 31 m
   - B. 40 m
   - C. 42 m
   - D. 51 m
   - E. 62 m

9. Snowy the Polar Bear fires a gun. The bullet starts from rest and has a velocity of $600\,\mathrm{m\,s^{-1}}$ when it emerges from a 1.50 m barrel. For how long is the bullet in the barrel? Assume constant acceleration.
   - A. 1.8 ms
   - B. 2.3 ms
   - C. 2.5 ms
   - D. 4.0 ms
   - E. 5.0 ms

10. It takes Snowy the Polar Bear two minutes to walk up a 20-m long stopped escalator. When the escalator is moving, it takes Snowy 65 seconds to be carried up the escalator when Snowy is standing still. How much time would it take Snowy to walk up the moving escalator?
    - A. 36 s
    - B. 42 s
    - C. 53 s
    - D. 74 s
    - E. 92 s

11. After Snowy the Polar Bear's escalator adventure, Snowy decides to use an elevator. The combined weight of the Snowy and the elevator cab is 27.8 kN. What is the tension in the cable if Snowy's speed is increasing at a rate of $1.22\,\mathrm{m\,s^{-1}}$?
    - A. 3.46 kN
    - B. 15.6 kN
    - C. 27.8 kN
    - D. 29.3 kN
    - E. 31.3 kN

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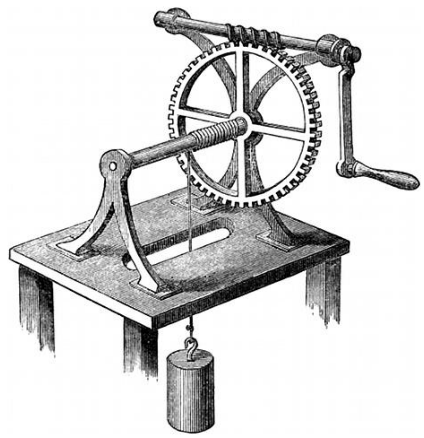
Questions 12-16 refer to Elena's lift, illustrated below. Her lift consists of a handle arm 12 inches long connected to a single started screw with a 3/4 inch lead, a 52-tooth gear connected to an axle 3 inches in diameter, and a 10 pound weight connected with a string wrapped around the aforementioned axle. She's hoping to patent it.



12. What type of gear system is being used in this lift?
    - A. Sun and planet gear
    - B. Spur gear system
    - C. Sliding train
    - D. Harmonic drive
    - E. Worm drive

13. Assuming the force is applied at the end of the handle arm, what is the ideal mechanical advantage of this lift system?
    - A. 139
    - B. 208
    - C. 312
    - D. 416
    - E. 1660

14. What is the diameter of the gear?
    - A. 12 in
    - B. 19 in
    - C. 25 in
    - D. 37 in
    - E. 40 in

15. What is the power required to spin the handle at 25 rpm?
    - A. $1.14 \times 10^{-4}$ hp
    - B. $6.29 \times 10^{-4}$ hp
    - C. $6.55 \times 10^{-4}$ hp
    - D. $7.30 \times 10^{-4}$ hp
    - E. $9.46 \times 10^{-4}$ hp

16. Elena is testing her lift by repeatedly lifting and lowering the weight. When doing so, she notices that changing directions of rotation often results in a moment where the mass is still but the screw keeps turning. What phenomenon is occurring?
    - A. Mesh reduction
    - B. Inertia
    - C. Deflection
    - D. Backlash
    - E. String elasticity

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17. Snowy the Polar Bear loves playing football. Snowy kicks the ball so that it lands 46 m away with a flight time of 4.5 s. What is the magnitude of the ball's initial velocity if the ball leaves Snow's foot 150 cm above ground?
    - A. $10\,\mathrm{m\,s^{-1}}$
    - B. $22\,\mathrm{m\,s^{-1}}$
    - C. $23\,\mathrm{m\,s^{-1}}$
    - D. $24\,\mathrm{m\,s^{-1}}$
    - E. $32\,\mathrm{m\,s^{-1}}$

18. Snowy the Polar Bear decides to throw a football off the cliff with an initial velocity of $15.0\,\mathrm{m\,s^{-1}}$ at an angle of 20.0° below the horizontal. What is the horizontal displacement 2.30 s later?
    - A. 28.1 m
    - B. 28.3 m
    - C. 30.5 m
    - D. 32.4 m
    - E. 37.7 m

19. Out of anger, Snowy the Polar Bear uses a hockey stick to propel a 3.5 kg organic chemistry textbook from rest on the floor. A constant, horizontal 25 N force is applied over a distance of 0.90 m, resulting in a speed of $1.60\,\mathrm{m\,s^{-1}}$. What is the coefficient of kinetic friction between the book and floor?
    - A. 0.56
    - B. 0.58
    - C. 0.69
    - D. 0.87
    - E. 0.89

20. What type of simple machine is the hockey stick that Snowy used to hit the organic chemistry textbook?
    - A. First Class Lever
    - B. Second Class Lever
    - C. Third Class Lever
    - D. Wedge
    - E. Screw

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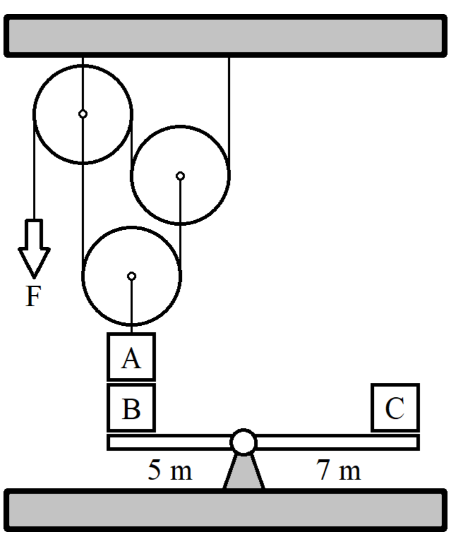
Questions 21-23 refer to Howard's compound machine, drawn below. He applies a force $F$ on the rope. Blocks $A$, $B$, and $C$ are 7 kg, 5 kg, and 6 kg, respectively. The width of the blocks are negligible. The lever and pulleys are massless and friction is negligible. The grey surfaces represent immovable surfaces.




21. How hard must Howard pull on the rope to keep the system in equilibrium?
    - A. 8.83 N
    - B. 11.8 N
    - C. 16.5 N
    - D. 35.3 N
    - E. 44.1 N

22. What power must Howard exert in order to lift block $A$ up 3 meters in 0.4 seconds?
    - A. 90 W
    - B. 206 W
    - C. 227 W
    - D. 515 W
    - E. 883 W

23. Once lifted up 3 meters, block $A$ is released from rest and accelerates downwards. Block $A$ then collides with block $B$ and all three blocks travel together with the lever. What is the speed of block $C$ right after the collision? Assume the lever is held still until right before the collision.
    - A. $0.63\,\mathrm{m\,s^{-1}}$
    - B. $2.63\,\mathrm{m\,s^{-1}}$
    - C. $3.16\,\mathrm{m\,s^{-1}}$
    - D. $8.29\,\mathrm{m\,s^{-1}}$
    - E. $9.08\,\mathrm{m\,s^{-1}}$

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24. Snowy the Polar Bear uses a cable to pull a box of fish across the floor. If the coefficient of kinetic friction between the box and floor is 0.35 and the maximum tension of the cable is 1100 N, what angle (with respect to the horizontal) should Snowy pull the box at to move the most amount of sand at a constant velocity?
    - A. 13°
    - B. 19°
    - C. 27°
    - D. 34°
    - E. 48°

25. What simple machine can be described as an inclined plane wrapped around a cylinder?

    A. Lever
    B. Inclined plane
    C. Pulley
    D. Wedge
    E. Screw

26. Snowy the Polar Bear is using a pulley system of IMA = 10 to lift a 1350 N piano a distance of 4 m. How much force does Snowy need to use?

    A. 34 N
    B. 135 N
    C. 193 N
    D. 256 N
    E. 338 N

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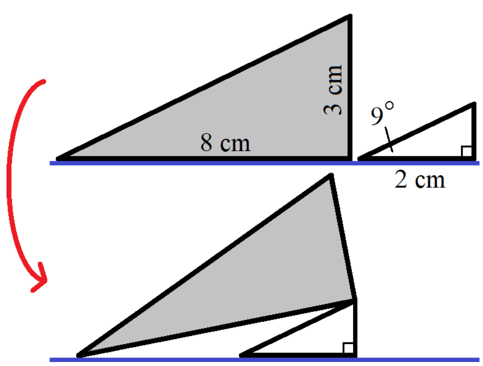
Questions 27-29 refer to Jeffrey's new and exciting inclined plane, drawn below. It consists of a 600 g inclined plane of uniform density and a wedge. The wedge is used to lift the plane such that the plane's edge meets the wedge's top edge. The top surface of the inclined plane has a coefficient of kinetic friction of 0.25. All other surfaces are frictionless.




27. What is the efficiency of the inclined plane before it is lifted?

    A. 20.9 %
    B. 25.7 %
    C. 45.1 %
    D. 60.0 %
    E. 87.6 %

28. What is the minimum horizontal force Jeffrey must apply to on the wedge to lift the inclined plane when the wedge is just touching it? Assume the plane's bottom left edge is fixed to the ground.

    A. 160 mN
    B. 173 mN
    C. 613 mN
    D. 622 mN
    E. 678 mN

29. What is the actual mechanical advantage of the inclined plane after it is lifted?

    A. 1.55
    B. 1.61
    C. 1.62
    D. 1.71
    E. 2.58

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30. There are three reversible (the input and output **can** be switched) simple machines with IMAs of 7, 9, and 10. How many possible unique IMA values can be made through the combinations of these three machines? At least two machines must be used.

    A. 1 to 5
    B. 6 to 10
    C. 11 to 15
    D. 16 to 25
    E. $\leq 26$

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## Section B: Free Response

**Points are shown for each question or sub-question, for a total of 90 points.**

1. (10 points) A double started screw is passed through a thin, stiff board such that it is upright. It has a pitch $P$, a mass $m$, a shaft radius $R$, and a cap diameter $D$. Wrapped around the screw cap is an elastic rope that acts like a spring by applying a force $kL$ at the edge of the cap, where $k$ is a constant and $L$ is the elongation of the rope. Finally, a mass $M$ is placed on the screw cap. Assume no friction between the screw and the board. Answer all sub-questions in terms of the given variables and other fundamental constants.
   (a) (3 points) What is the mechanical advantage of the screw? *(Note: You will have to make an assumption.)*
   (b) (4 points) How many revolutions will the screw make before it reaches equilibrium? Initially $L$ is equal to 0.
   (c) (3 points) The answer to the previous question assumed that the rope's thickness is negligible. Would the number of revolutions increase or decrease if the rope's thickness is considered? Explain. Assume the rope continues to wrap around the screw cap.

2. (10 points) A chisel is modeled by an isosceles triangular prism with a vertex angle of $\theta$ and a mass $m$. A hammer is modeled by a block of mass $M$. The chisel is lined up perpendicular to the side of a stone block and hit by the hammer traveling horizontally at velocity $V$. Subject only to the stone's reaction force, the two are predicted to come to rest after traveling together over a distance $D$. Answer all sub-questions in terms of the given variables and other fundamental constants.
   (a) (2 points) What is the ideal mechanical advantage of the chisel?
   (b) (3 points) Determine the average horizontal reaction force from the stone block. Assume the same ideal conditions in the problem statement.
   (c) (4 points) The chisel and hammer end up travelling a distance $d$, where $d < D$. Using the previous force term, find the resultant energy efficiency of the interaction by comparing the initial energy to the work done.
   (d) (1 point) Identify a potential source of energy loss other than friction.

3. (25 points) As we cannot conduct the device testing portion of the event, you will draft up a design of a device, which should be able to determine a mass ratio up to 21:2. It **must** consist of an **inclined plane** and a **wheel and axle**, connected together by a **pulley system with an IMA of 2**.

   (a) (2 points) Give a one sentence explanation of your device design.

   (b) (12 points) Draw **two device diagrams**.

      i. Diagram 1:

         1. View is from an angle so it is 3-dimensional.
         2. Major features are labeled (i.e. simple machine types, mass locations, etc.)

      ii. Diagram 2:

         1. View is from the side so it is 2-dimensional.
         2. Important dimensions are labeled (i.e. simple machine dimensions, device base width, etc.)

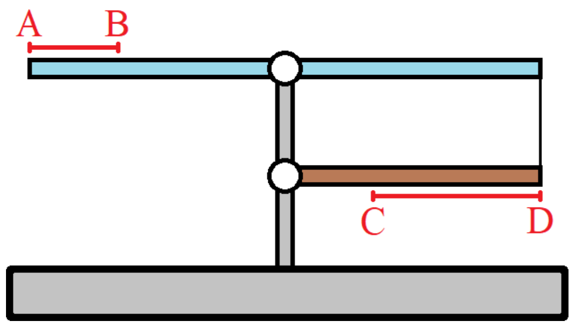
   (c) (8 points) You are given **3 blocks** with known masses: mass A = 630 g, mass B = 810 g, and mass C = 60 g. You will place two pairs of masses (A&B and A&C, so two masses at a time) on your device such that it is in equilibrium. For each pair of masses:

      1. Draw a simple diagram with the location of the masses indicated.
      2. Work through the appropriate calculations to show the device is balanced.

      You should have **two diagrams** and **two sets of calculations**.

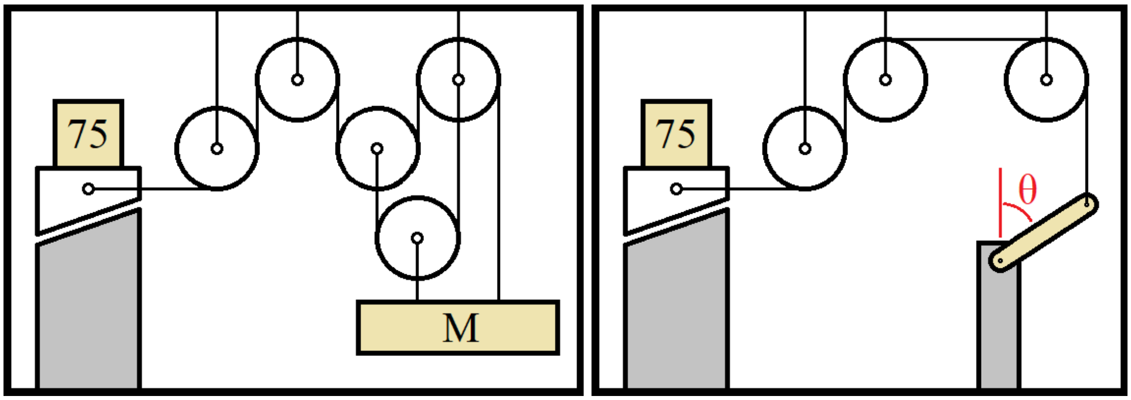
   (d) (3 points) Let's assume there is a 5 % error in the angle of the inclined plane. For the second mass pair (A&C), what is the maximum mass ratio (A/C) that could be estimated from your design? Show your work.

4. (25 points) In preparation for the device testing portion of the event, Phoebe acquires a (pretty much) massless, frictionless, indestructible lever arm that is 1 m long. However, the morning of the competition, her partner reminds her that the device must be a compound lever system. Panicking, she decides to use a 50 cm long tree branch she finds on the ground as another lever. Shown below is a diagram of her device with the bottom beam being the tree branch. The two beams are connected by a lightweight string. The red ranges AB and CD are 20 cm and 35 cm long. The grey surfaces are immovable surfaces.




   (a) (7 points) Let's first assume the mass of the tree branch is negligible and the system is as ideal as can be.
      i. (3 points) If Phoebe can only hang masses in the red ranges (defined by points A, B, C, and D), what is the maximum and minimum mass ratios she can determine?
      ii. (4 points) If masses only hang in the red ranges, what is the maximum reaction force at the fulcrum of the bottom beam, in N? The device must be in equilibrium. *(Hint: Each mass must be between 20.0 and 800.0 g.)*
   (b) (10 points) Unfortunately, since Phoebe has to use the tree branch (which has mass), she needs to collect some information about this non-ideal material. She has two masses X and Y, which are 600 g and 125 g, respectively. She hangs mass X at point A and mass Y at point C and records an instantaneous counterclockwise angular acceleration of $10.4\,\mathrm{rad\,s^{-2}}$. She then hangs mass Y at point B and mass X at point D and records an instantaneous clockwise angular acceleration of $13.7\,\mathrm{rad\,s^{-2}}$. The tree branch can be approximated as a uniform rod.
      i. (5 points) What is the frictional torque acting at the fulcrum of the bottom beam, in N m?
      ii. (5 points) What is the mass of the tree branch, in kg?
   (c) (8 points) Finally, Phoebe needs to figure out the best measurement strategy to get the optimal total score. She estimates that it takes 5 seconds for her to halve the percent error of one mass ratio estimate. *(Note: She must spend the entire 5 seconds each time.)* Assume she starts at 100 percent error for both mass ratio estimates. There are two mass ratios. Recall the formulas for the Time Score (TS) = ((240 − total time spent in seconds) / 240) × 15 points and Ratio Scores (R1 and R2) = (1 − (abs(AR − MR) / AR)) × 15 points, where AR is the actual mass ratio and MR is the estimated mass ratio. The total score is the TS + R1 + R2, making the theoretical maximum total score 45.
      i. (4 points) How many seconds should she spend measuring the masses?
      ii. (4 points) To three significant figures, what is the expected optimal total score?

5. (20 points) Shown below is a diagram of a compound machine system. The inclined plane is at a 15 degree angle, with respect to the horizontal, and a massless, trapezoidal wedge of the same angle is placed on it. The coefficient of static and kinetic friction between the two is 0.15 and 0.10. Fixed on the wedge is a 75 kg mass.




(a) (9 points) The cord is passed through a pulley system and attached to a plate of mass $M$, shown in the left diagram. The pulleys are massless and frictionless.

i. (2 points) What is the ideal mechanical advantage of the pulley system?

ii. (4 points) What is the maximum and minimum $M$ such that the system is in equilibrium, in kg?

iii. (3 points) Let $M$ be 300 kg. If the plate starts from rest, what is its displacement after 9 seconds, in m? Assume the plate does not hit the ground and the wedge stays on the inclined plane.

(b) (11 points) The cord is now threaded through a simpler pulley system and connected to a rod at an angle $\theta$ with respect to the vertical, shown in the right diagram. For simplicity, assume the cord is always tangential to the rod. The rod is 4 meters long.

i. (1 point) Let the applied force on the rod be through the cord. What class lever best describes the rod?

ii. (4 points) What are the maximum and minimum angles (from 0 to 180 degrees) for the system to be in equilibrium, in degrees? Assume a linear density of $15\,\mathrm{kg\,m^{-1}}$.

iii. (6 points) The minimum angle for the system to be in equilibrium can be represented as the function $\theta_{min}(\lambda)$, where $\lambda$ is the linear density of the rod, in $\mathrm{kg\,m^{-1}}$. Find this function and identify its limits.