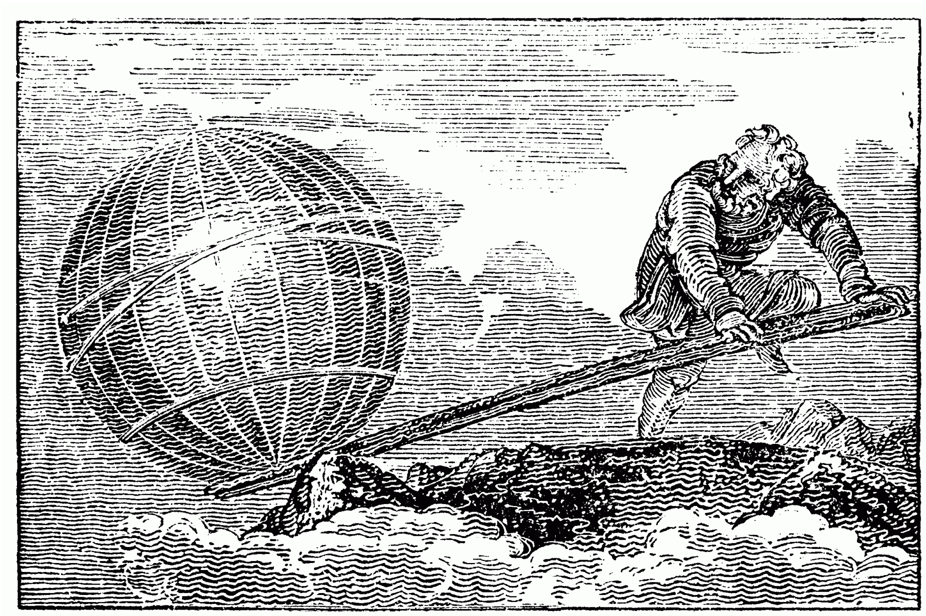
Science Olympiad
Machines C
Duke Invitational

January 30, 2021



Directions:

- Each team will be given **50 minutes** to complete the test.
- There are two sections: **§A** (Multiple Choice) and **§B** (Free Response).
- For significant figures, **use 3 or more in your answers** unless otherwise specified.
- Whenever needed, take the acceleration of gravity, g, to be $9.81\,\mathrm{m\,s^{-2}}$.
- Tiebreakers, in order: §B, §B2, §B4, §A1, . . . , §A30.
- Best of luck! And may the odds be ever in your favor.

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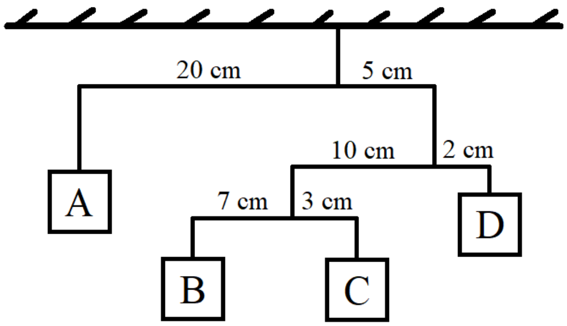
Feedback? Test Code: *2021DUSO-MachinesC-Silo*

Section A: Multiple Choice

Each question in this section is worth two points, for a total of 60 points.

Questions 1-4 refer to Oliver's mobile, shown below. The mobile is currently in equilibrium. Horizontal lines represent rigid, ideal rods. Vertical lines represent massless ropes. Object D has a mass of 50 kg.



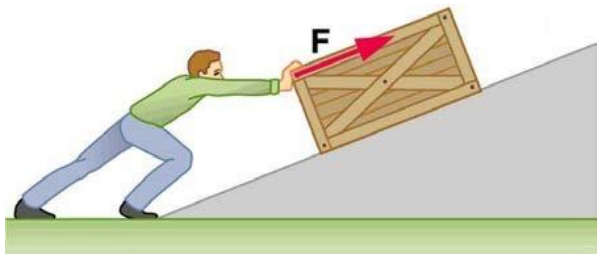
1. What is the mass of object A?
   A. 2 kg
   B. 3 kg
   C. 7 kg
   D. 10 kg
   E. 15 kg

2. What is the mass of object B?
   A. 2 kg
   B. 3 kg
   C. 7 kg
   D. 10 kg
   E. 15 kg

3. What is the mass of object C?
   A. 2 kg
   B. 3 kg
   C. 7 kg
   D. 10 kg
   E. 15 kg

4. What is the tension in the rope connected to the ceiling?
   A. 698 N
   B. 736 N
   C. 1172 N
   D. 1359 N
   E. 1444 N

Questions 5-9 refer to Chris pushing a box up a ramp, depicted below. The ramp has a height of 2 m and a slant length of 10 m. The box has a mass of 10 kg.



5. Suppose the ramp is ideal. What is its IMA?
   A. 0.2
   B. 4
   C. 5
   D. 10
   E. 20

6. If Chris pushes with a force of 30 N, what is the net acceleration of the box?

   A. $0.2\,\mathrm{m\,s^{-2}}$

   B. $1.0\,\mathrm{m\,s^{-2}}$

   C. $2.0\,\mathrm{m\,s^{-2}}$

   D. $3.0\,\mathrm{m\,s^{-2}}$

   E. $6.8\,\mathrm{m\,s^{-2}}$

7. Chris pushes the box until it has a velocity of $4\,\mathrm{m\,s^{-1}}$ up the ramp, then releases it. How high (vertically) above the release point does the box reach before sliding back down?

   A. 0.16 m

   B. 0.60 m

   C. 0.82 m

   D. 1.2 m

   E. 1.6 m

8. Suppose the box-ramp surface now has a coefficient of static friction 0.3 and the block is at rest. How much force is needed to push the box up the ramp?

   A. 20 N

   B. 29 N

   C. 30 N

   D. 48 N

   E. 98 N

9. The coefficient of kinetic friction between the box and the ramp is 0.2. What is the efficiency of the ramp (for uphill motion)?

   A. 20%

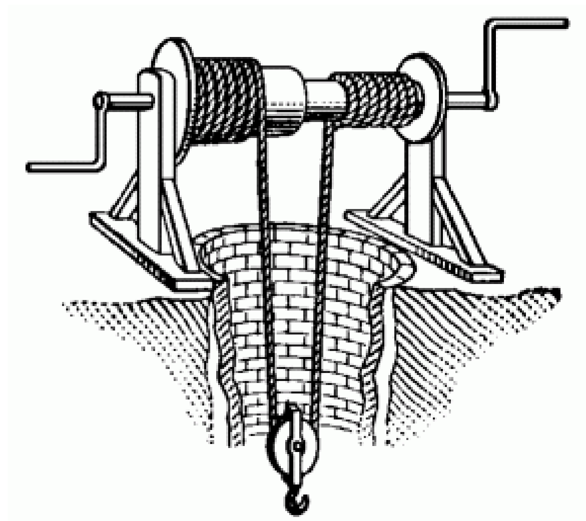
   B. 40%

   C. 50%

   D. 80%

   E. 100%

Questions 10-16 refer to Howard's differential windlass, depicted below. The larger (left) and smaller (right) axles have a diameter of 16 in and 12 in, respectively. The handle (the radial part) has a length of 24 in and the ropes are massless. The hook is attached to an also massless bucket (not depicted) containing an unknown volume of water.



10. If the water bucket is raised, what is the rotation vector direction of the axle?

    A. Up

    B. Down

    C. Left

    D. Right

    E. Out (of the page)

11. Which machine(s) are used in this windlass? (Select all that apply)

    A. Pulley

    B. Lever

    C. Wedge

    D. Wheel and Axle

    E. Screw

12. What is the IMA of the windlass?
    A. 12
    B. 18
    C. 24
    D. 36
    E. 48

13. If a full bucket exerts a torque of 1.87 N m onto the handle, what is the volume of the bucket? *(Note: 1 g = 1 cm^3)*
    A. $0.0075\,\mathrm{m}^3$
    B. $0.0150\,\mathrm{m}^3$
    C. $0.0183\,\mathrm{m}^3$
    D. $0.0220\,\mathrm{m}^3$
    E. $0.0439\,\mathrm{m}^3$

14. If Howard exerts 40 W of power over 20 seconds to lift the bucket from the bottom to the top of the 40 ft well, how much water is lifted? *(Note: 1 g = 1 ml)*
    A. 6.69 L
    B. 7.24 L
    C. 9.03 L
    D. 9.77 L
    E. 10.5 L

15. The rope has a maximum tensile strength of 2000 N. Find an expression of the maximum upwards acceleration of the bucket, if it contains a mass M of water. Let up be positive.
    A. $(4000 - g)/M$
    B. $(M - 2000)/g$
    C. $(2000 + g)/M$
    D. $(4000 - Mg)/M$
    E. $(2000/M) - g$

16. Which of the following changes would increase the efficiency of the windlass? (Select all that apply)
    A. Reinforce the well's paneling.
    B. Buy a lighter bucket.
    C. Lubricate the axle.
    D. Pack rope windings closer together.
    E. Use a stronger rope.

Questions 17-21 refer to Arnold's splitting axe, depicted below. The blade has a mass of 2 kg and a constant slope, measuring 1 cm at the widest and 10 cm long. Arnold places his left hand at the base of the handle, 90 cm from the blade, and his right hand 60 cm from the blade.

17. If Arnold's left hand remains stationary as the axe is swung, what is the IMA of the entire axe?
    A. 1.5
    B. 3.3
    C. 6.7
    D. 10
    E. 15

18. In the scenario above, what machine(s) are being used? (Select all that apply)
    - A. 1st class lever
    - B. 2nd class lever
    - C. 3rd class lever
    - D. Inclined plane
    - E. Wedge

19. Suppose Arnold holds the axe horizontally, with the blade resting on a block of wood. If 200 N of tensile (output) force is required to split the wood, how much effort force is needed?
    - A. 20 N
    - B. 60 N
    - C. 100 N
    - D. 200 N
    - E. 600 N

20. Now Arnold instead swings the axe at the wood, and the blade is moving at $5\,\mathrm{m\,s^{-1}}$ at the moment of contact. Arnold does not apply any force during contact, and the collision is fully inelastic. If the blade comes to a stop after 0.1 seconds, what is the average force exerted by the wood on the axe?
    - A. 10 N
    - B. 50 N
    - C. 60 N
    - D. 100 N
    - E. 200 N

21. In this swing, the blade makes a cut 2 cm deep in the wood block. What is the total amount of work done by the wood on the axe?
    - A. 2 J
    - B. 5 J
    - C. 10 J
    - D. 20 J
    - E. 25 J

Questions 22-26 refer to Patrice applying a wrench to a screw, shown below. The single-start screw has 32 threads per inch and a shaft diameter of 3/16 in. Her wrench (from the pivot point to her hand) is 10 in long.

22. What is the IMA of the wrench-screw system?
    - A. 18.8
    - B. 107
    - C. 320
    - D. 1710
    - E. 2010

23. If Patrice turns the wrench at a constant rate, with her hand moving a distance of 7 inches over 0.3 seconds, what is the linear speed of the screw?[1]
    - A. 0.011 in/s
    - B. 0.073 in/s
    - C. 0.21 in/s
    - D. 0.96 in/s
    - E. 2.3 in/s

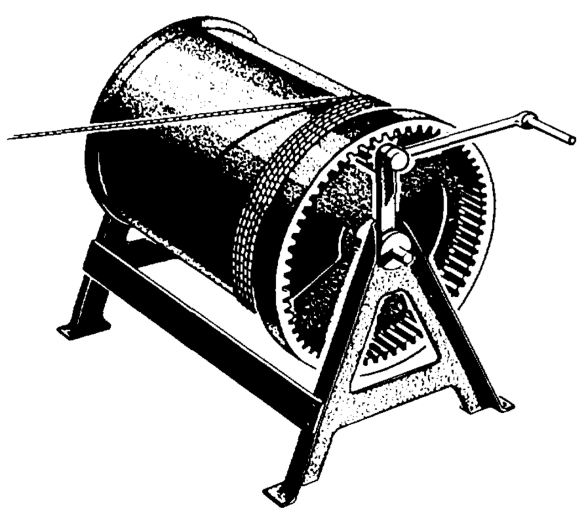
24. Patrice exerts a torque of $4\,\mathrm{lbf \cdot ft}$ as he turns the wrench. What is the input power being exerted on the screw?
    - A. $4.7\,\mathrm{lbf \cdot ft/s}$
    - B. $7.8\,\mathrm{lbf \cdot ft/s}$
    - C. $8.4\,\mathrm{lbf \cdot ft/s}$
    - D. $9.3\,\mathrm{lbf \cdot ft/s}$
    - E. $13\,\mathrm{lbf \cdot ft/s}$

[1] This question was removed from scoring due to an error. It is now fixed.

25. Screws are useful as fasteners because of the self-locking property. If the screw is self-locking, which choice(s) must be true? (Select all that apply)
    A. Efficiency is less than 50%
    B. Efficiency is equal to 50%
    C. Efficiency is greater than 50%
    D. The screw cannot convert linear motion to rotational motion
    E. The screw cannot be loosened

26. Suppose the screw is self-locking, and an output force of at least 1000 lbf is required (along the screw axis). What is the minimum force Patrice must apply to the wrench?
    A. 0.497 lbf
    B. 1.00 lbf
    C. 2.01 lbf
    D. 4.69 lbf
    E. 9.38 lbf

Questions 27-30 refer to Alice the sailor's cable winch, depicted below. The crank arm is 75 cm long and the drum has a diameter of 50 cm. The small pinion gear has 10 teeth and the larger gear has 100 teeth. One end of the cable is tied to a 700 kg anchor and the other end is glued onto the drum.



27. What is the IMA of the cable winch?
    A. 50
    B. 30
    C. 20
    D. 15
    E. 10

28. Assume a sailor can only crank with a force of 100 N. In order to lift the anchor, how many **more** sailors does Alice need and what direction should the crank be turned?
    A. 1; counterclockwise
    B. 2; clockwise
    C. 3; clockwise
    D. 3; counterclockwise
    E. 4; counterclockwise

29. The anchor must be raised! What is the average power exerted by each sailor to raise the anchor from a depth of 120 m in 30 minutes?[2]
    A. 458 W
    B. 305 W
    C. 229 W
    D. 153 W
    E. 102 W

30. When the sailors begin to lift the anchor, there are only two and a half rotations of rope around the drum. The coefficient of static friction between the drum and the rope is 0.4. What is the maximum force the glued end of the rope must be able to withstand so that the rope doesn't slip?
    A. 303 N
    B. 297 N
    C. 272 N
    D. 13.1 N
    E. 12.8 N

[2]This question was removed from scoring due to an error. It is now fixed.

Section B: Free Response

Points are shown for each question or sub-question, for a total of 90 points.

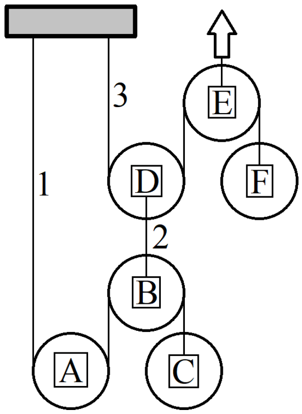
1. (10 points) A double-started screw with a 5 mm pitch is used to move a press up and down. This screw is rotated with a 40 cm rod of steel, with one end welded to the screw and the other end where force is acted upon. The press is lowered when the rod is rotated counterclockwise. Welded on the press is a metal wedge in the form of a long isosceles triangle prism with a 5° angle.
   (a) (1 point) Is the screw right-handed or left-handed?
   (b) (4 points) A log is placed under the press. The log splits after receiving a force of 15 000 N. If the edge of the rod is pulled with a force of 12 N, what is the efficiency of the machine?
   (c) (3 points) If the rod made 20 full rotations, how much energy was lost, in J?
   (d) (2 points) If the metal wedge is replaced with a metal cone, how would the IMA of the machine change? Explain your answer.

2. (30 points) As we cannot conduct the device testing portion of the event, you will draft up a design of a device. The device will follow the event and construction parameters and must be able to determine a mass ratio up to 9:1. However, it **must** consist of a **class 2 lever** connected to a **class 3 lever**. You are also allowed to use **one fixed pulley** in your design.
   (a) (1 point) Give a one sentence explanation of your device design.
   (b) (12 points) Draw two device diagrams.
      i. One diagram must be an isometric view of the device with labeled features.
      ii. One diagram must be a profile of the device with proper dimensions.
   (c) (8 points) Thoroughly explain the testing process for two mass ratios: 9:1 and 7:3. Each mass ratio explanation must include a profile diagram with the mass locations indicated and must work through the appropriate calculations.
   (d) (9 points) In reality, mechanical devices are never ideal. Answer the following questions by considering the scenario below and applying it to your design.

      You test your device using two unknown masses. Based off of your (ideal) calculations, you determine that the masses have a mass ratio of 7:1. However, when you weigh the masses, you find the masses are 200 g *and* 25 g.

      i. (1 point) What is the percent error in your mass ratio estimate?
      ii. (4 points) Assuming the **sole source of error** was from a friction torque acting at the point of rotation of the **class 2 lever**, calculate the magnitude of this torque, in Nm. Show your work.
      iii. (4 points) Another possible source of error could have been the misalignment of the center of mass and the point of rotation of the **class 3 lever**. Assuming that this was the **sole source of error**, calculate the magnitude of this "misalignment" torque, in Nm. Show your work.

3. (20 points) A pulley system in static equilibrium is shown to the left with ideal, variable-mass pulleys, and the effort force is denoted by the arrow. Pulley A has a mass of 2 kg and pulley F has a mass of 5 kg. The grey block is an immovable surface.



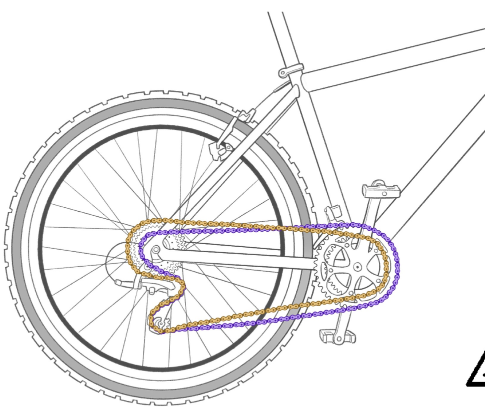
(a) (3 points) Find the mass of pulley C and the tension in rope 3, in N.

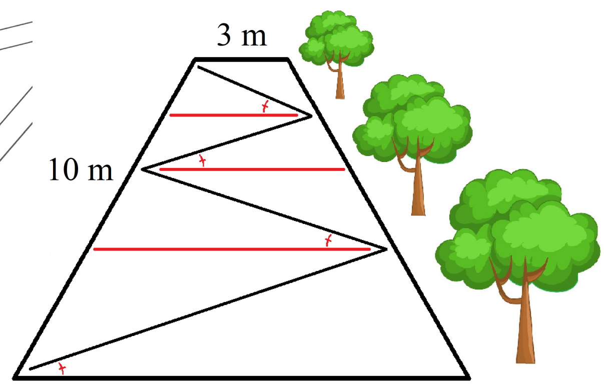
(b) (6 points) If the sum of the masses of all 6 pulleys is considered the load, what is the AMA (in terms of as few unknowns as possible)?

(c) (5 points) If the mass of pulley D is 3 times the mass of pulley B, what is the tension in rope 2, in N?

(d) (6 points) The effort force is increased so that pulley D accelerates upward at $6\,\mathrm{m\,s^{-2}}$. Find the acceleration (magnitude and direction) of pulley A, in $\mathrm{m\,s^{-2}}$.

4. Bicycles are designed to have various gearing ratios to let the cyclist adjust the IMA of a bicycle depending on the situation. The bicycle shown below on the left diagram has two gear sets. The set on the back tire has 3 gears: 40-tooth, 32-tooth, and 20-tooth. The set on the pedal has 2 gears: 35-tooth and 14-tooth. The pedals are 30 cm from each other and the back tire has a 31.1 cm radius.





(a) (3 points) List all possible gearing ratios of the bicycle, in the form (x:1).
(Hint: The gear ratio is the ratio between the rotation of the driver gear vs. the driven gear.)

(b) (3 points) The bicycle is driven up a 40° incline on the lowest gear (easiest to climb uphill). What is the IMA of the bicycle + incline system?

(c) (6 points) Anna the cyclist weighs 60 kg and can output 200 W.

i. (4 points) Anna rides the bicycle on the lowest gear up the same 40° incline. Assuming the bicycle is massless, its components are frictionless, and the wheels roll without slipping, what is the maximum constant speed Anna can cycle at, in $\mathrm{m\,s^{-1}}$?

ii. (2 points) Anna switches gears to the highest one. How does her cadence (pedaling rate) change? Does it increase, decrease, or stay the same? Explain why.

(d) (18 points) Anna's friend, Benjamin, is a cycling enthusiast, but cycles recreationally. He is 70 kg, can output 125 W, and only bikes in the lowest gear.

i. (3 points) Even though his power output is less than Anna's, he still wants to ride the bike quickly. He decides to zig-zag four times up a 10 m stretch of the incline, 3 m wide, shown above in the right diagram. The four angles shown are all congruent, with the red horizontal lines being lines of constant elevation. What is this angle, in degrees?

ii. (3 points) Calculate the force he must exert tangentially to the pedal gear to travel along the zig-zag path at a constant speed, in N. Assume that each turn is instantaneous and no speed is lost.

iii. (12 points) Assume the bicycle is in its lowest gear. The IMA of the bicycle + incline system can be represented as a function IMA(n), where n is the number of zig-zags. For example, IMA(0) is equal to the answer in (b) and IMA(4) is equal to the IMA used in (d.ii). This function can be represented in the form $a(n^b + c)^d$. Find a, b, c, and d.