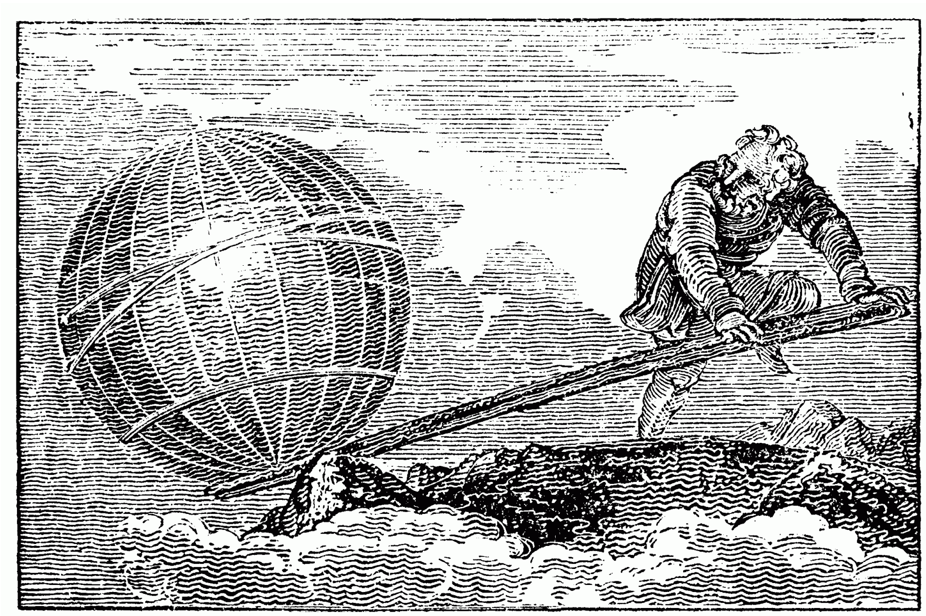
Science Olympiad
Machines C
Duke Invitational

January 30, 2021



Section B Solutions

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Feedback? Test Code: *2021DUSO-MachinesC-Silo*

Section B: Free Response

Points are shown for each question or sub-question, for a total of 90 points.

1. (10 points) A double-started screw with a 5 mm pitch is used to move a press up and down. This screw is rotated with a 40 cm rod of steel, with one end welded to the screw and the other end where force is acted upon. The press is lowered when the rod is rotated counterclockwise. Welded on the press is a metal wedge in the form of a long isosceles triangle prism with a 5° angle.

   (a) (1 point) Is the screw right-handed or left-handed?

   (b) (4 points) A log is placed under the press. The log splits after receiving a force of 15 000 N. If the edge of the rod is pulled with a force of 12 N, what is the efficiency of the machine?

   (c) (3 points) If the rod made 20 full rotations, how much energy was lost, in J?

   (d) (2 points) If the metal wedge is replaced with a metal cone, how would the IMA of the machine change? Explain your answer.

Solution:

(a) Most screws are right-handed and follow the convention of "righty-tighty and lefty-loosey". Since the press is lowered when the rod if rotated counterclockwise, the screw is reversed, so we know the screw is left-handed.

(b) We can apply the formula for efficiency (AMA/IMA) and solve for AMA and IMA.

$$\begin{cases} \text{AMA} = \frac{15\,000\,\text{N}}{12\,\text{N}} = 1250 \\ \text{IMA} = \text{IMA}_{\text{screw}} \times \text{IMA}_{\text{wedge}} = \frac{2\pi \times 40\,\text{cm}}{1\,\text{cm}} \times \frac{1}{2\tan(5°/2)} = 2880 \end{cases} \implies \frac{\text{AMA}}{\text{IMA}} \times 100\,\% = 43.4\,\%$$

(c) The energy lost is defined by the percent energy inefficiency multiplied by the word done.

$$(1-\eta) \times W = 0.566 \times 12\,\text{N} \times 20 \times 2\pi \times 0.4\,\text{m} = 341\,\text{J}$$

(d) The IMA of the machine would increase as a cone has a smaller surface area, increasing the force it exerts in order to balance the system. However, as the size of the wedge is not specified the IMA of the machine could also decrease. The answer is accepted if the explanation mentions a greater/lesser surface area with the cone and has an appropriate justification.

2. (30 points) As we cannot conduct the device testing portion of the event, you will draft up a design of a device. The device will follow the event and construction parameters and must be able to determine a mass ratio up to 9:1. However, it **must** consist of a **class 2 lever** connected to a **class 3 lever**. You are also allowed to use **one fixed pulley** in your design.

   (a) (1 point) Give a one sentence explanation of your device design.

   (b) (12 points) Draw two device diagrams.

      i. One diagram must be an isometric view of the device with labeled features.

      ii. One diagram must be a profile of the device with proper dimensions.

   (c) (8 points) Thoroughly explain the testing process for two mass ratios: 9:1 and 7:3. Each mass ratio explanation must include a profile diagram with the mass locations indicated and must work through the appropriate calculations.

   (d) (9 points) In reality, mechanical devices are never ideal. Answer the following questions by considering the scenario below and applying it to your design.

   > *You test your device using two unknown masses. Based off of your (ideal) calculations, you determine that the masses have a mass ratio of 7:1. However, when you weigh the masses, you find the masses are* 200 g *and* 25 g.

      i. (1 point) What is the percent error in your mass ratio estimate?

      ii. (4 points) Assuming the **sole source of error** was from a friction torque acting at the point of rotation of the **class 2 lever**, calculate the magnitude of this torque, in Nm. Show your work.

      iii. (4 points) Another possible source of error could have been the misalignment of the center of mass and the point of rotation of the **class 3 lever**. Assuming that this was the **sole source of error**, calculate the magnitude of this "misalignment" torque, in Nm. Show your work.

Solution:

(a) Basic description of the device is given in one sentence.

(b) (4 points) Device consists of two of the specified machines.

(1 point) Diagram is isometric.

(3 points) Isometric diagram properly labels major features in the device features in the device.

(1 point) Diagram is in profile.

(2 points) Dimensions for major features are included and device fits within size restrictions.

(c) (2 points) Each diagram with mass locations depicted.

(2 points) Each correct calculation for the mass ratio.

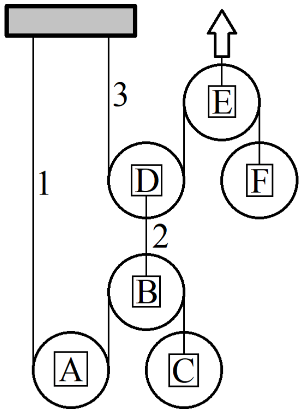
(d) i. The mass ratio is 8:1 and using the percent error formula ($|(v_A - v_E)/v_E| \times 100\,\%$, where v_A is the actual value observed and v_E is the expected value) we get the answer.

$$\left|\frac{7-8}{8}\right| \times 100\,\% = 12.5\,\%$$

ii. Answers vary. Explanation identifies the location of the torque and works through the appropriate calculations to reach the correct answer.

iii. Answers vary. Explanation identifies the location of the torque and works through the appropriate calculations to reach the correct answer.

3. (20 points) A pulley system in static equilibrium is shown to the left with ideal, variable-mass pulleys, and the effort force is denoted by the arrow. Pulley A has a mass of 2 kg and pulley F has a mass of 5 kg. The grey block is an immovable surface.



(a) (3 points) Find the mass of pulley C and the tension in rope 3, in N.

(b) (6 points) If the sum of the masses of all 6 pulleys is considered the load, what is the AMA (in terms of as few unknowns as possible)?

(c) (5 points) If the mass of pulley D is 3 times the mass of pulley B, what is the tension in rope 2, in N?

(d) (6 points) The effort force is increased so that pulley D accelerates upward at $6\,\mathrm{m\,s^{-2}}$. Find the acceleration (magnitude and direction) of pulley A, in $\mathrm{m\,s^{-2}}$.

Solution:

(a)

$$2T_1 - m_A g = 0 \implies T_1 = \frac{1}{2}m_A g = 9.8\,\mathrm{N}$$
$$T_1 - m_C g = 0 \implies m_C = \frac{T_1}{g} = \frac{1}{2}m_A = 1\,\mathrm{kg}$$
$$T_3 - m_F g = 0 \implies T_3 = m_F g = 49\,\mathrm{N}$$

(b)

$$T_2 - 2T_1 - m_B g = 0 \implies m_B g = T_2 - 2T_1$$
$$2T_3 - T_2 - m_D g = 0 \implies m_D g = 2T_3 - T_2$$

$$\begin{aligned} F_{out} &= m_A g + m_B g + m_C g + m_D g + m_E g + m_F g \\ &= m_A g + \frac{1}{2}m_A g + 2T_3 - 2T_1 + m_E g + m_F g = m_E g + 157\,\mathrm{N} \end{aligned}$$
$$F_{in} - 2T_3 - m_E g = 0 \implies F_{in} = 2T_3 + m_E g = m_E g + 98\,\mathrm{N}$$
$$\frac{F_{out}}{F_{in}} = \frac{m_E g + 157\,\mathrm{N}}{m_E g + 98\,\mathrm{N}}$$

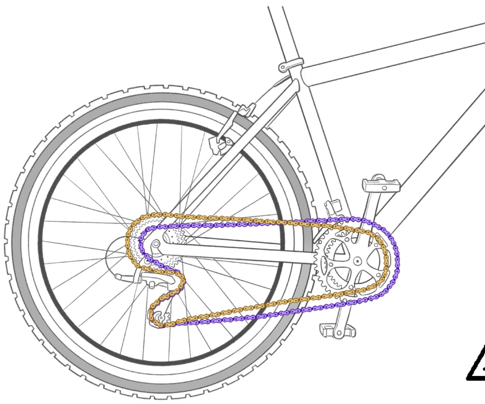
(c)

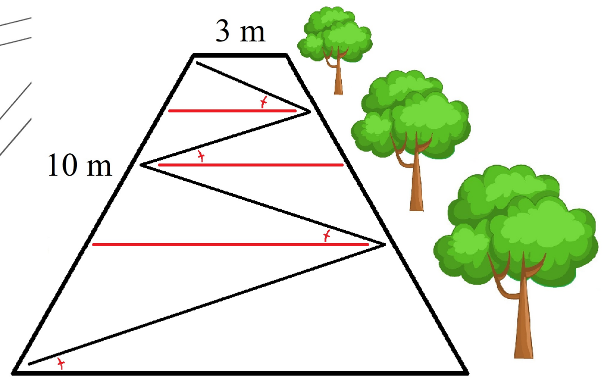
$$m_B g = T_2 - 2T_1 \text{ and } m_D g = 2T_3 - T_2 \text{ from (b)}$$
$$3m_B g = m_D g \implies 3(T_2 - 2T_1) = 2T_3 - T_2 \implies 3T_2 - 6T_1 = 2T_3 - T_2$$
$$T_2 = \frac{1}{2}(T_3 + 3T_1) = 39.2\,\mathrm{N}$$

(d)

$$a_B = a_D = 6\,\mathrm{m\,s^{-2}}$$
$$2T_1 - m_A g = m_A a_A \implies T_1 = \frac{1}{2}m_A(a_A + g)$$
$$T_1 - m_C g = m_C a_C \implies \frac{1}{2}m_A(a_A + g) = T_1 = m_C(a_C + g) \implies a_A = a_C$$
$$-2a_A + 2a_B - a_C = 0 \implies -3a_A + 2a_B = 0$$
$$a_A = \frac{2}{3}a_B = 4\,\mathrm{m\,s^{-2}}, \text{ upwards}$$

4. Bicycles are designed to have various gearing ratios to let the cyclist adjust the IMA of a bicycle depending on the situation. The bicycle shown below on the left diagram has two gear sets. The set on the back tire has 3 gears: 40-tooth, 32-tooth, and 20-tooth. The set on the pedal has 2 gears: 35-tooth and 14-tooth. The pedals are 30 cm from each other and the back tire has a 31.1 cm radius.





(a) (3 points) List all possible gearing ratios of the bicycle, in the form (x:1).

(b) (3 points) The bicycle is driven up a 40° incline on the lowest gear (easiest to climb uphill). What is the IMA of the bicycle + incline system?

(c) (6 points) Anna the cyclist weighs 60 kg and can output 200 W.

i. (4 points) Anna rides the bicycle on the lowest gear up the same 40° incline. Assuming the bicycle is massless, its components are frictionless, and the wheels roll without slipping, what is the maximum constant speed Anna can cycle at, in $\mathrm{m\,s^{-1}}$?

ii. (2 points) Anna switches gears to the highest one. How does her cadence (pedaling rate) change? Does it increase, decrease, or stay the same? Explain why.

(d) (18 points) Anna's friend, Benjamin, is a cycling enthusiast, but cycles recreationally. He is 70 kg, can output 125 W, and only bikes in the lowest gear.

i. (3 points) Even though his power output is less than Anna's, he still wants to ride the bike quickly. He decides to zig-zag four times up a 10 m stretch of the incline, 3 m wide, shown above in the right diagram. The four angles shown are all congruent, with the red horizontal lines being lines of constant elevation. What is this angle, in degrees?

ii. (3 points) Calculate the force he must exert tangentially to the pedal gear to travel along the zig-zag path at a constant speed, in N. Assume that each turn is instantaneous and no speed is lost.

iii. (12 points) Assume the bicycle is in its lowest gear. The IMA of the bicycle + incline system can be represented as a function IMA(n), where n is the number of zig-zags. For example, IMA(0) is equal to the answer in (b) and IMA(4) is equal to the IMA in (d.ii). This function can be represented in the form $a(n^b + c)^d$. Find a, b, c, and d.

Solution:

(a) Trying all possible gear pairing gets us $3 \times 2 = 6$ possible gear ratios. Partial credit is given to unreduced or flipped ratios.

(b) To calculate the overall IMA, find the product of IMAs from the three simple machines: the inclined plane, the gearing ratio, and the wheel and axle. One point given for each IMA.

$$\text{IMA} = \text{IMA}_{\text{incline}} \cdot \text{IMA}_{\text{gear}} \cdot \text{IMA}_{\text{wheel}} = \frac{1}{\sin(40°)} \cdot 2.86 \cdot \frac{15\,\text{cm}}{31.1\,\text{cm}} = 2.15$$

(c) i. Find the vertical velocity of the bicycle with the given power. Then, with the vertical velocity, calculate the speed using the incline angle. Partial credit is given to $0.729\,\text{m}\,\text{s}^{-1}$, the speed that her feet move at on pedals.

$$P = mgv \implies v = \frac{P}{mg} = \frac{200\,\text{W}}{60\,\text{kg} \cdot 9.81\,\text{m}\,\text{s}^{-2}} = 0.340\,\text{m}\,\text{s}^{-1}$$

$$s = \frac{v}{\sin(40°)} = 0.529\,\text{m}\,\text{s}^{-1}$$

ii. Her cadence decreases. Since her power stays the same and the bicycle's IMA decreases, she must apply a greater force with a lower speed.

(d) i. The crux of the question involves taking advantage of the "reflection" that occurs at each turn by extending the incline out horizontally. This means the path taken by the bicycle is just a straight line, with each turn being reflected over the edge of the incline. In the case of the first question, we can extend the incline by 9 m to create a 10 m by 12 m rectangle. To get the angle, we can just use the inverse tangent function: $\tan^{-1}(10/12) = 39.8°$.

ii. Find the overall IMA again, but with the new distance. Then, divide Benjamin's weight by the IMA to get the force he needs to exert.

$$\text{IMA} = \text{IMA}_{\text{incline}} \cdot \text{IMA}_{\text{gear}} \cdot \text{IMA}_{\text{wheel}} = \frac{\sqrt{12^2 + 10^2}}{10\sin(40°)} \cdot 2.86 \cdot \frac{15\,\text{cm}}{31.1\,\text{cm}} = 3.35$$

$$\frac{70\,\text{kg} \cdot 9.81\,\text{m}\,\text{s}^{-2}}{3.35} = 205\,\text{N}$$

iii. We can use the same IMA formula from (d.ii) and rearrange to find the constant coefficients.

$$\text{IMA}(n) = \frac{\sqrt{(3n)^2 + 10^2}}{10\sin(40°)} \cdot 2.86 \cdot \frac{15\,\text{cm}}{31.1\,\text{cm}} = 0.214\sqrt{9n^2 + 100} = 0.643(n^2 + 11.1)^{0.5}$$

$$a = 0.643,\ b = 2,\ c = 11.1,\ d = 0.5$$

Out of all of the questions I've written this season, this one was definitely my favorite. So, I would like to give a shoutout to Allison Eto and Riana Santos from the team Iolani Artichokie, who were the only ones who got this question right. Congratulations to them!