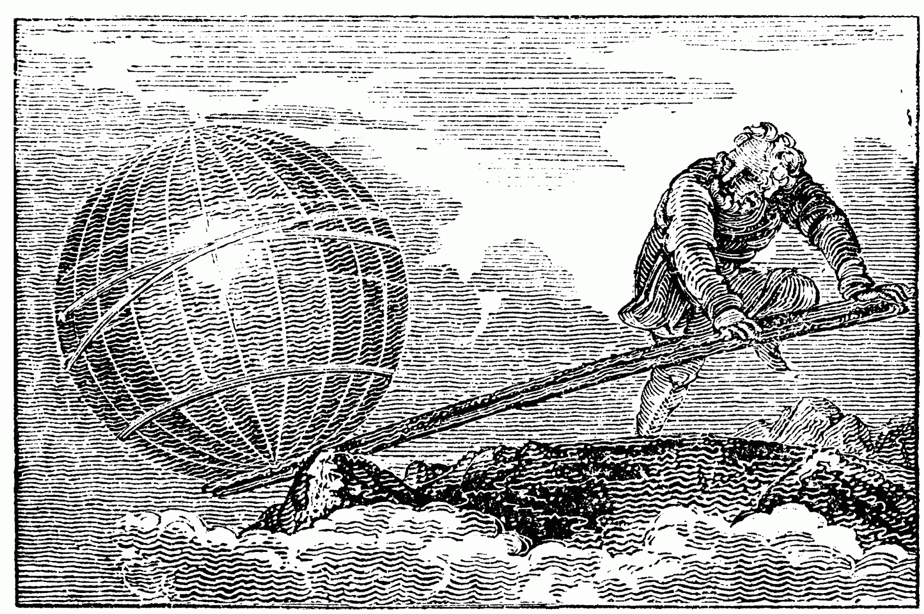
Science Olympiad
Machines C
Golden Gate Invitational

February 13, 2021



Section B Solutions

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Feedback? **Test Code**: *2021GGSO-MachinesC-Cable*

Section B: Free Response

Points are shown for each question or sub-question, for a total of 90 points.

2. (30 points) As we cannot conduct the device testing portion of the event, you will draft up a design of a device, which should be able to determine a mass ratio up to 12:1. It **must** consist of **two out of the three** following simple machines: **wheel and axle** (must use two), **wedge** (the angle is adjustable), or **pulley** (can use up to two pulleys, fixed or moveable).

   (a) (2 points) Give a one sentence explanation of your device design.

   (b) (12 points) Draw **two device diagrams**.

      i. Diagram 1:

         1. View is from an angle so it is 3-dimensional.
         2. Major features are labeled (i.e. simple machine types, mass locations, etc.)

      ii. Diagram 2:

         1. View is from the side so it is 2-dimensional.
         2. Important dimensions are labeled (i.e. simple machine dimensions, device base width, etc.)

   (c) (8 points) You are given 3 known masses: mass A = 600 g, mass B = 50 g, and mass C = 60 g. You will place two pairs of masses (A&B and B&C, so **two masses at a time**) on your device such that it is in equilibrium. For each pair of masses:

      1. Draw a simple diagram with the location of the masses indicated.
      2. Work through the appropriate calculations to show the device is balanced.

      You should have **two diagrams** and **two sets of calculations**.

   (d) (8 points) Listed below are three potential sources of error, each corresponding to a simple machine. Separately consider **two sources of error**. Explain how each would affect your device (How does it change your device? How could it increase and/or decrease the mass ratio?) and provide a possible solution to remove or diminish the error.

      1. Wheel and axle: non-uniform (nor radially-uniform) wheel density
      2. Wedge: error in the wedge angle
      3. Pulley: axial friction

Solution:

(a) (2 points) Basic description of the device is given in one sentence.

(b) (4 points) Device consists of two of the specified machines.

(1 point) Diagram is isometric.

(3 points) Isometric diagram properly labels major features in the device features in the device.

(1 point) Diagram is a side view.

(3 points) Dimensions for major features are included and device fits within size restrictions.

(c) For each mass ratio:

(2 points) Diagram with mass locations depicted.

(2 points) Shows proper work with correct calculations to show that the device is in equilibrium.

If device does not use the correct machines, points are halved.

(d) For each source of error:

(2 points) Explains how it would affect mass ratio (shows how the ratio would *increase* or *decrease*).

(2 point) Provides a reasonable method to minimize the error.

3. (12 points) A spring is attached to the bottom of a first class lever at a distance d to the right of the lever's fulcrum. A uniform cube of mass M is placed at a distance of D on the left of the fulcrum, resulting in the spring elongating by $D/10$. The force applied by this spring is given by kx, where k is the spring constant, and x is the displacement of the spring. Assume the lever beam is massless and the fulcrum is frictionless.

(a) (3 points) In terms of M, g, and d, find the spring constant.

(b) (2 points) Now, a sphere of mass $3M$ is placed somewhere on the lever. The spring relaxes by $D/30$ (meaning it is still elongated by some amount). Is the new mass placed on the same side as the spring, or the same side as the cube? Explain.

(c) (4 points) In terms of D, how far from the fulcrum is the sphere?

(d) (3 points) Now, the sphere is moved to the other side of the fulcrum, but remains at the same distance from the fulcrum. In terms of D, what is the new total elongation of the spring (relative to when the spring has no load on it)?

Solution:

(a) Balancing torques, we get:

$$\Sigma\tau = MgD - k\frac{D}{10}d = 0 \implies k = \frac{10Mg}{d}$$

(b) Since the spring is less elongated, it exerts a smaller force, and thus we know the sphere must be "helping" the spring, and so it is on the same side as the spring.

(c) Let l be the distance we want to find. Balancing torques again, we get:

$$\Sigma\tau = MgD - kxd - 3Mgl = MgD - \frac{10Mg}{d}\left(\frac{D}{10} - \frac{D}{30}\right)d - 3Mgl = 0 \implies l = \frac{D}{9}$$

(d) Once more, we balance torques:

$$\Sigma\tau = MgD + 3Mg\frac{D}{9} - \frac{10Mg}{d}xd = 0 \implies x = \frac{2D}{15}$$

4. (12 points) A block of mass M is pushed up a frictionless curve, where the angle of inclination slowly increases with horizontal distance.

   For parts (a), and (b), assume that you apply a force F (where $F < Mg$) on the block such that the force is maximally effective in pushing the block up the curve. For parts (c) and (d), assume that you apply a force P that is always horizontal.

(a) (1 point) In which direction is the force F exerted on the block?

(b) (3 points) What is the largest angle of inclination that the block can reach when the force F is applied? Give your answer in terms of M, F, and fundamental constants, as appropriate.

(c) (4 points) What is the largest angle of inclination that the block can reach when the force P is applied? Give your answer in terms of M, P, and fundamental constants, as appropriate.

(d) (4 points) How much work will have been done on the block when it can no longer move any further? Give your answer in terms of M, P, and fundamental constants, as appropriate. If impossible to answer, explain why.

Solution:

(a) Parallel to the line tangent to the curve where the block is located at that time. This can be reasoned as the weight force perpendicular to the line tangent to the curve will be opposed by the normal force from the surface. That means any force acting in that direction is "wasted" and we only need to oppose the force parallel to the surface of the curve.

(b) Balance forces along the direction of the curve using $\Sigma F = 0$:

$$\Sigma F = F - Mg\sin\theta = 0 \implies \theta = \sin^{-1}\left(\frac{F}{Mg}\right)$$

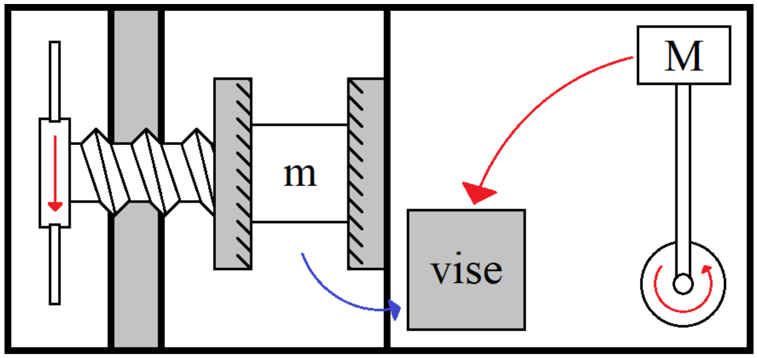
(c) Same as (b):

$$\Sigma F = P\cos\theta - Mg\sin\theta = 0 \implies \theta = \tan^{-1}\left(\frac{P}{Mg}\right)$$

(d) Not enough information.

You don't know what height the block will be at when it reaches the maximum angle of inclination. If you were given a function of how the angle changes with distance (horizontal or vertical), you could use it to solve for the final height.

5. (24 points) The diagram below depicts a vise system and your friend's proprietary design of her Smash-O-Matic™ (patent pending). Assume all components are frictionless and ideal unless otherwise specified.



(a) (10 points) On the left of the diagram is your vise, consisting of a single-start screw with a 5 cm screw cap diameter and a 5 mm lead. The screw also has two rods, each 30 cm long, welded onto the edge of the screw cap by their ends. A block of mass m ($m = 8$ kg) is placed in the vise and tightened between two identical rough surfaces.

i. (2 points) To use the vise, you apply a force at the end of the rods and a force is exerted by the plate onto the mass m. What is the IMA of the vise?

ii. (2 points) Most vises (and screws) are self-locking. What would be the maximum AMA of your vise if it is self-locking? *(Note: Do not use this AMA for the following sub-questions.)*

iii. (3 points) You apply a force of 10 N on the end of a rod to hold the block still. If it takes a downwards force of 120 N to move the block, calculate the coefficient of static friction between the block and the rough surfaces.

iv. (3 points) What is the minimum upwards force to move the same block, in N?

(b) (14 points) On the right of the diagram is the Smash-O-Matic™, which consists of a hammer, with a 1 kg head and a 40 cm handle, connected to a motor that outputs a constant torque of 3 Nm. The "vise" square represents a side profile of the clamped block. The hammer begins upright and accelerates until it's horizontal. Then, the head is released from the handle and immediately hits the block. The hammer head and block collide perfectly inelastically and travel together, until they come to rest. Assume the handle mass is negligible, the block and hammer head stay in the vise, and the same, constant 10 N force is applied to the vise.

i. (3 points) Find the velocity of the hammer head right before it hits the block, in $\mathrm{m\,s^{-1}}$.

ii. (3 points) Find the velocity of the hammer head right after it hits the block, in $\mathrm{m\,s^{-1}}$.

iii. (3 points) Determine the energy efficiency of the collision, in %.

iv. (5 points) Given the coefficient of kinetic friction is 0.015, how far does the block travel before it comes to rest, in cm?

Solution:

(a) i. One rotation of the screw is $(50\,\text{mm} + 300\,\text{mm})\pi$, which moves the screw a distance of $5\,\text{mm}$. Dividing the two gives: $\text{IMA} = 2042\,\text{mm}/5\,\text{mm} = 408$.

ii. The maximum efficiency where a machine is self-locking is $50\,\%$. This means the maximum AMA is half of the IMA: $\text{AMA}_{max} = 408/2 = 204$.

iii. Drawing a free-body diagram, we tally a total of four forces: a downwards weight force of $(8\,\text{kg} \cdot 9.81\,\text{m}\,\text{s}^{-2})$ $78.5\,\text{N}$, a downwards applied force of $120\,\text{N}$, a normal force on both sides of the block $(408 \cdot 10\,\text{N})$ $4080\,\text{N}$, and an upwards frictional force of $F_f = 2 \cdot 4080\,\text{N} \cdot \mu_s$. *(Note that F_f is multiplied by 2 as there are two friction surfaces.)* Using $\Sigma F = 0$, we get:

$$\Sigma F = 78.5\,\text{N} + 120\,\text{N} - 2 \cdot 4080\,\text{N} \cdot \mu_s = 0 \implies \mu_s = 0.0243$$

iv. Let P be the applied force. Using the same setup, we just reverse the direction of the frictional force and do a similar calculation:

$$\Sigma F = 78.5\,\text{N} - P + F_f = 0 \implies P = 78.5\,\text{N} + 2 \cdot 4080\,\text{N} \cdot 0.0243 = 277\,\text{N}$$

(b) i. Using energy methods, we add together the work done by the motor, W_m, and the gravitational potential energy, U_g, to get the final velocity, v_f. Let M be the mass of the hammer head and l be the length of the handle.

$$\frac{1}{2}Mv_f^2 = W_m + U_g = \tau\theta + Mgl \implies v_f = \sqrt{2\left(\frac{\tau\theta}{M} + gl\right)} = 4.16\,\text{m}\,\text{s}^{-1}$$

ii. This collision is a perfectly inelastic one with both masses moving together after the collision. Using conservation of momentum, we get

$$Mv_i = (m+M)v_f \implies v_f = Mv_i/(m+M) = 0.462\,\text{m}\,\text{s}^{-1}$$

iii. The energy efficiency of a process is the initial energy E_i divided by the final energy E_f. Using this, we get:

$$E_i = \frac{1}{2}Mv_i^2,\ E_f = \frac{1}{2}(m+M)v_f^2 = \frac{1}{2}(m+M)\left(\frac{Mv_i}{m+M}\right)^2 = \frac{1}{2}\frac{M^2}{(m+M)}v_i^2$$

$$\frac{E_i}{E_f} \times 100\,\% = \frac{M}{m+M} \times 100\,\% = 11.1\,\%$$

iv. Finally, we do one last energy calculation, where the initial energy is from the v_f we found in (b.ii), the energy dissipated from friction is $F_f d$, and the gravitational potential energy is $(m+M)gd$. Equating these two and solving for d, we get:

$$\frac{1}{2}(m+M)v_f^2 - F_f d + (m+M)gd = 0 \implies d = \frac{(m+M)v_f^2}{2(F_f - (m+M)g)}$$

$$d = \frac{9\,\text{kg} \cdot (0.462\,\text{m}\,\text{s}^{-1})^2}{2(2 \cdot 4080\,\text{N} \cdot 0.015 - 9\,\text{kg} \cdot 9.81\,\text{m}\,\text{s}^{-2})} = 2.80\,\text{cm}$$