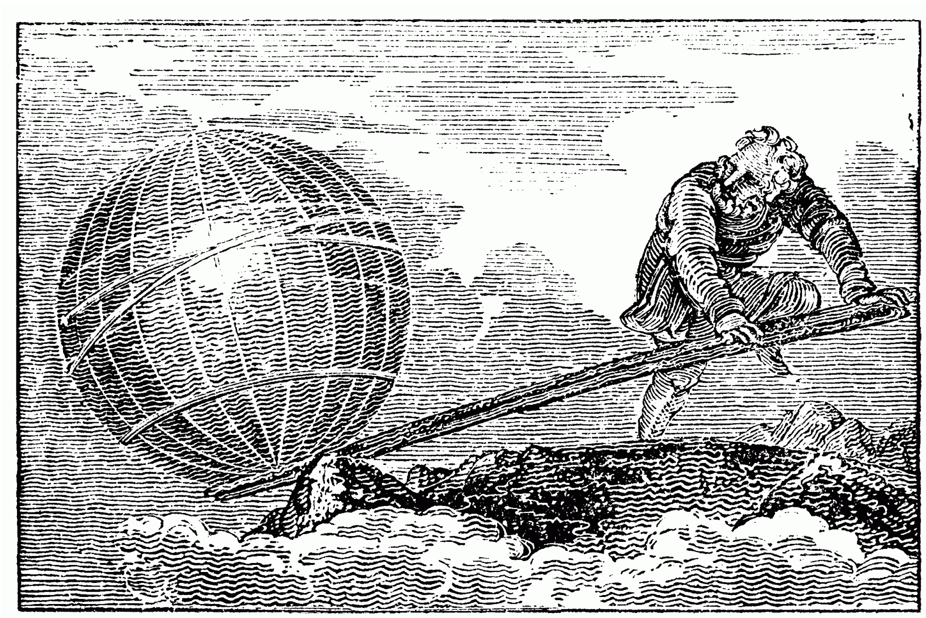
Science Olympiad
Machines C
Mira Loma Invitational

January 9, 2021



Section B Solutions

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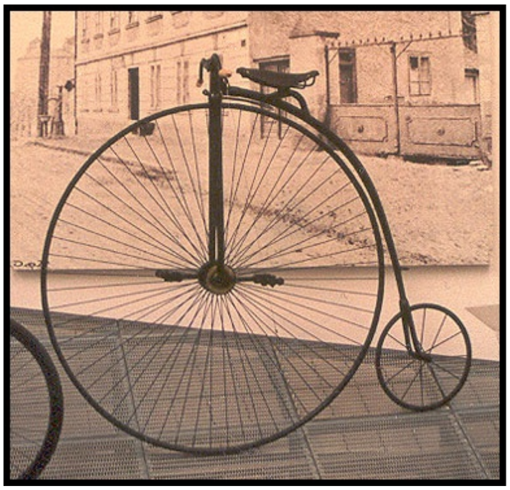
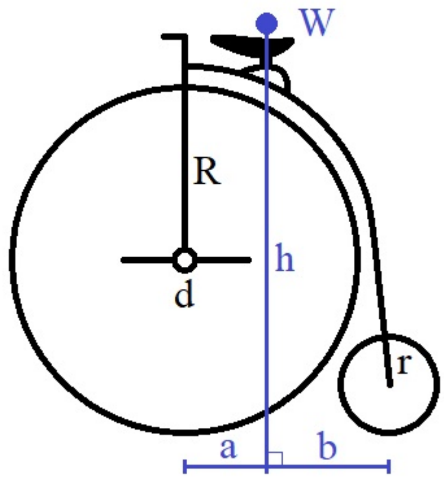
Feedback? **Test Code**: *2021MLSO-MachinesC-Chain*

Section B: Free Response

Points are shown for each question or sub-question, for a total of 90 points.

1. (18 points) Below is a picture and diagram of a penny-farthing, which was the first machine to be called a bicycle. It does not use a gear train, but rather is equipped with a large wheel to provide a mechanical advantage to its rider. The variables in the diagram are as follows: **R**, the radius of the large wheel; **r**, the radius of the small wheel; **d**, the distance between the pedals; **W**, the weight of the passenger; **h**, the minimum distance between the passenger's center of mass and the ground; and **a** and **b**, the distances from the point on the ground closest to the passenger's center of mass to the ground contact point of the large wheel and the small wheel, respectively.





(a) (1 point) What simple machine best describes the penny-farthing?

(b) (2 points) Give a possible reason for why the penny-farthing used such a large wheel.

(c) (15 points) The following questions will be answered in terms of the variables in the diagram.

i. (2 points) Calculate the IMA of the penny-farthing. Assume the system is ideal and massless.

ii. (3 points) Let F_R and F_r represent the normal forces on the large and small wheel on level ground. Determine the equations that satisfy the equilibrium conditions.

iii. (3 points) Using the previous equations, solve for F_R and F_r.

iv. (3 points) Given c_l as the coefficient of rolling friction with dimension of length and F_{in} as the input force on the pedal. Find the AMA of the penny-farthing only in terms of R, d, W, c_l, and F_{in}. Simplify the expression given $2a = b$ and $R = 3r$. *(Use F_R and F_r for partial credit)*

v. (4 points) One downside of the penny-farthing is the precarious location of its center of mass. Riders often fall head-first if they go up or down too steep of an incline. Find the maximum angle of descent and maximum angle of ascent.

Solution:

(a) Wheel and axle

(b) Accepted answers include: lowers IMA, allows for higher speed of travel, smoother ride on rough surfaces, etc. Many teams incorrectly mentioned a higher IMA. Make sure to pay attention to where the input and output forces are in a machine.

(c) i. Let F_{in} be the force exerted onto the pedal and F_{out} be the friction from the road acting on the edge of the wheel. By balancing torques around the wheel center, we find the IMA (F_{out}/F_{in}).

$$F_{in} \times \frac{d}{2} = F_{out} \times R \implies \frac{F_{out}}{F_{in}} = \frac{d}{2R}$$

ii. The equilibrium conditions mentioned are the balance of forces and torques. Since forces on the penny-farthing only act vertically and torques can be calculated around the central point on the ground, we get the following equations.

$$\Sigma F = F_R + F_r - W = 0 \implies F_R + F_r = W$$
$$\Sigma \tau = F_R \times a - F_r \times b = 0 \implies aF_R = bF_r$$

iii. With this system of two equations, we now solve for F_R and F_r.

$$F_R = \frac{bW}{a+b};\ F_r = \frac{aW}{a+b}$$

iv. Now we bring everything together. AMA is defined similarly to IMA as F_{out}/F_{in}. To find F_{out}, we combine the input force with the frictional forces on both wheels. Note that since c_l has dimensions of length, we divide by the radius of the wheels to get out coefficient of friction.

$$F_{out} = \Sigma F = F_{in}\frac{d}{2R} - F_R\frac{c_l}{R} - F_r\frac{c_l}{r} = \frac{F_{in}d - 2c_l(F_R + 3F_r)}{2R}$$

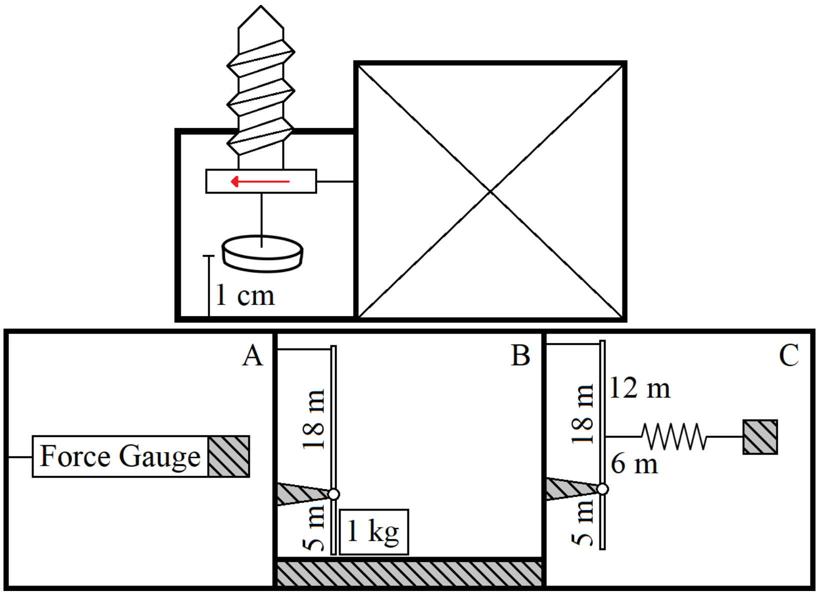
$$AMA = \frac{F_{out}}{F_{in}} = \frac{1}{F_{in}} \times \frac{F_{in}d - 2c_l(F_R + 3F_r)}{2R} = \frac{d}{2R} - \frac{c_l(F_R + 3F_r)}{F_{in}R}$$

Now plugging $2a = b$ for F_R and F_r from (c.iii), we get $F_R = 2W/3$ and $F_r = W/3$ which we use to get our final answer.

$$AMA = \frac{d}{2R} - \frac{5Wc_l}{3F_{in}R}$$

v. For the penny-farthing to tip over, the weight of the passenger must be directly above the points where torque is applied, which are the two ground contact points. Construct a right triangle using the segments of length a and h. To make the previous assertion true, the triangle must be rotated counterclockwise until its hypotenuse is vertical. Working through the geometry, this angle (the angle of descent) is the same as the angle between the hypotenuse and segment h. Using trigonometric identities, we get $\theta_{desc} = \tan^{-1}(a/h)$ ($90° - \tan^{-1}(h/a)$ also accepted). The same logic is applied again to find θ_{asc}.

2. (30 points) Shown below is a diagram of the system (not to scale). A single-threaded screw of a 9 mm pitch, 2 mm shaft radius, and a 3 cm cap radius is fully embedded into the ceiling. A 2 kg disk is hung to the center of the screw, 1 cm off the ground. A cord is wrapped around the screw cap and connected to three setups at the bottom: A, B, and C. Assume all components are frictionless unless specified.



(a) (8 points) The cord is connected to a force gauge, shown in diagram A.

i. (2 points) Find the IMA of the screw. The input force is applied by the cord.

ii. (3 points) The force gauge measures a force of 1 N. What is the AMA of the system?

iii. (3 points) Calculate the screw's cap radius assuming the AMA is the IMA, in cm.

(b) (15 points) The cord is now disconnected from the force gauge and is connected to the end of a massless lever, acting as the effort force. At the end of the lever is a 1 kg block resting on a low friction surface ($\mu_k = 0.05$). The setup is shown in diagram B. *(Note: Use the IMA found in (a.i) for all calculations, not the force given in (a.ii).)*

i. (1 point) Identify the lever class.

ii. (1 point) Calculate the IMA of the lever.

iii. (3 points) The system is currently barely at rest. Find the coefficient of static friction (μ_s).

iv. (4 points) The block is nudged into motion. Determine the time it takes for the disk to reach the ground, in s. Assume tension is constant and the block and lever remains in contact over the entire interval.

v. (6 points) The block is nudged into motion. How far does the block travel, in cm? Make sure to include all assumptions made in your calculation.

(c) (7 points) The block is disconnected and the disk is raised back to its original position. A spring of spring constant $50\,\mathrm{N\,m^{-1}}$ is connected to the lever at a different position, shown in diagram C. *(Note: Use the IMA found in (a.i) for all calculations, not the force given in (a.ii).)*

i. (1 point) Identify the lever class.

ii. (3 points) The disk is now slowly lowered until the system is in equilibrium. How far is it lowered, in cm?

iii. (3 points) What if the setup was on the Moon? How far would the disk be lowered, in cm?

Solution:

(a) i. Equate the work in and out for one screw revolution and use the definition of IMA (F_{out}/F_{in}).

$$W_{in} = W_{out} \implies F_{in} \times 2\pi \times 30\,\text{mm} = F_{out} \times 9\,\text{mm} \implies IMA = \frac{F_{out}}{F_{in}} = 20.9$$

ii. Once again, use the AMA definition (F_{out}/F_{in}).

$$AMA = \frac{F_{out}}{F_{in}} = \frac{2\,\text{kg} \times 9.81\,\text{m}\,\text{s}^{-2}}{1\,\text{N}} = 19.6$$

iii. We rearrange the equation in (a.i) to solve for radius and evaluate.

$$\frac{F_{out}}{F_{in}} = 19.6 = \frac{2\pi r}{0.9\,\text{cm}} \implies r = 2.81\,\text{cm}$$

(b) i. Since the fulcrum is between the two loads, it is a first class lever.

ii. $IMA = 18\,\text{m}/5\,\text{m} = 3.60$

iii. Since the block is at rest, we know $\Sigma F = 0$ and the forces are balanced by static friction.

$$2\,\text{kg} \times 9.81\,\text{m}\,\text{s}^{-2} \times \frac{3.6}{20.9} = 1\,\text{kg} \times 9.81\,\text{m}\,\text{s}^{-2} \times \mu_s \implies \mu_s = 0.344$$

iv. We can't assume a value for the tension in the string, so we have to set up the system of equations to constrain the setup. Let m_d, m_b, a_d, and a_b be the mass and acceleration of the disk and block. Also let T be the tension in the cord connected to the disk.

$$\begin{cases} m_d a_d = m_d g - T \\ m_b a_b = \frac{3.60}{20.9} T - m_b g \mu_k \\ 20.9 a_d = 3.60 a_b \end{cases} \implies a_d = 0.468\,\text{m}\,\text{s}^{-2}$$

From here, we can treat this as a kinematics problem with constant acceleration.

$$d = \frac{1}{2} a t^2 \implies t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2 \times 0.01\,\text{m}}{0.468\,\text{m}\,\text{s}^{-2}}} = 0.207\,\text{s}$$

v. Various energy conservation assumptions are applied (refer to key). Equate gravitational potential energy with frictional energy loss to find distance traveled.

$$m_d g h = m_b g \mu_k d \implies d = \frac{m_d h}{m_b \mu_k} = 40\,\text{cm}$$

(c) i. The force is on the same side and further from the fulcrum than the load, so it's 2nd class.

ii. Set up forces and solve for d when they are balanced.

$$m_d g \frac{3.00}{20.9} = k \left(\frac{20.9}{3.00} d\right) \implies d = \frac{2\,\text{kg} \times 9.81\,\text{m}\,\text{s}^{-2}}{50\,\text{N}\,\text{m}^{-1}} \left(\frac{3.00}{20.9}\right)^2 \times \frac{100\,\text{cm}}{1\,\text{m}} = 0.805\,\text{cm}$$

iii. The Moon's gravitational acceleration is 1/6 of Earth's, so we divide (c.ii) by 6 to get 0.134 cm.

3. (30 points) As we cannot conduct the device testing portion of the event, you will draft up a design of a device. The device will follow the event and construction parameters and must be able to determine a mass ratio up to 10:1. However, it **must** consist of one **inclined plane** (the angle is adjustable) along with **one out of the two** following simple machines: one out the two (can only use one) or one out the two (must use two pulleys, fixed or moveable).

(a) (1 point) Give a one sentence explanation of your device design.

(b) (12 points) Draw two device diagrams.

i. One diagram must be an isometric view of the device with labeled features.

ii. One diagram must be a profile of the device with proper dimensions.

(c) (10 points) Thoroughly explain the testing process for two mass ratios: 10:1 and 5:3. Each mass ratio explanation must include a profile diagram with the mass locations indicated and must work through the appropriate calculations.

(d) (3 points) By using an inclined plane, the design faces the issue of self-locking. What coefficient of friction is required for your device to work for the largest mass ratio, 10:1. Show your work.

(e) (4 points) Consider the following scenario:

> *After testing the device, you remove the masses, reset the device, and begin to pack it up. However, you notice the (lever begins/pulleys begin) to rotate on (its/their) own until (it comes/they come) to rest. You test this a few times and observe that (it/they) consistently (comes/come) to rest in the same position.*

Explain why this phenomenon occurs, how it would affect your mass ratio estimates, and provide a possible solution to remove or diminish it.

Solution:

(a) Basic description of the device is given in one sentence.

(b) (4 points) Device consists of two of the specified machines.

(1 point) Diagram is isometric.

(3 points) Isometric diagram properly labels major features in the device features in the device.

(1 point) Diagram is in profile.

(3 points) Dimensions for major features are included and device fits within size restrictions.

(c) (2 points) Each diagram with mass locations depicted.

(3 points) Each correct calculation for the mass ratio.

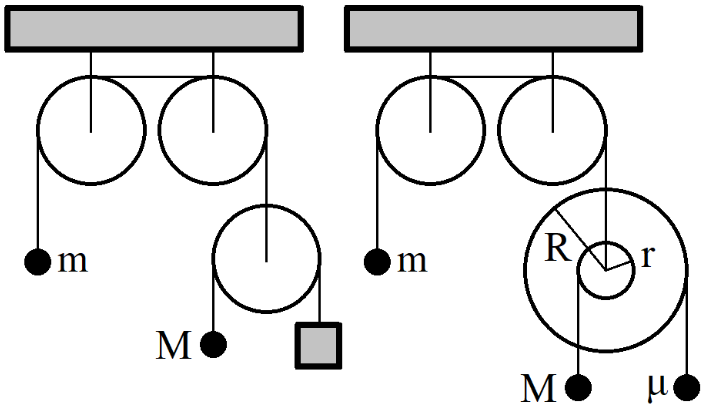
(d) To receive full points for this question, response must recognize the static friction must never be greater than the sliding force along the plane ($mg \sin \theta$). If it was greater, it would be impossible to determine the mass ratio as no force would be exerted to the other simple machine. Responses vary, but answers generally solved for the force equilibrium parallel to the plane to get a $\tan \theta$ term.

(e) (2 points) Determines the source of the error: center of mass and rotation axis are misaligned.

(1 point) Explains how it would affect mass ratio (shows how the ratio would *increase* or *decrease*).

(1 point) Provides a reasonable method to minimize the error.

4. (12 points) Shown below on the left is a pulley system of two point masses (m and M) and three massless pulleys, one movable and two fixed pulleys, connected by ideal strings that move without slipping. The grey boxes are immovable surfaces.



(a) (1 point) What is the IMA of the pulley system?

(b) (3 points) If $m = 5\,\text{kg}$ and $M = 20\,\text{kg}$, find the direction and magnitude of acceleration for mass M, in $\text{m}\,\text{s}^{-2}$.

The pulley system is modified. The movable pulley is switched with a coaxially connected ($r = 27\,\text{cm}$ and $R = 84\,\text{cm}$) one and a mass μ is added. The two mass-pulley pairs are connected by strings with fixed ends at the respective pulley. The new system is shown above on the right.

(c) (3 points) If $m = 60\,\text{kg}$, find M and μ so that the system is in equilibrium, in kg.

(d) (5 points) Find the tension in the string connected to mass μ, in N, when $M = 25\,\text{kg}$ and $\mu = 10\,\text{kg}$.

Solution:

(a) Both 2 and 1/2 were accepted as input force is not specified. The force from the string on mass m is double that on mass M.

(b) Set up system of equations from $\Sigma F = ma$ on each mass and string conservation. Let a_m and a_M be the acceleration of the respective masses and T be the tension in the string on mass M.

$$\begin{cases} ma_m = 2T - mg \\ Ma_M = T - Mg \\ 2a_m = -a_M \end{cases} \implies a_M = -8.08\,\mathrm{m\,s^{-2}}$$

(c) To reach equilibrium, the forces and torques on the movable pulley must be in equilibrium. We get the following equations: $M + \mu = m$ and $Mr = \mu R$. Solving, we get $M = 45.4\,\mathrm{kg}$ and $\mu = 14.6\,\mathrm{kg}$.

(d) Solution by April Cheng.

We apply a similar ideas from (b) and (c): set up $\Sigma F = ma$ on each mass, balance relative forces and torques around the movable pulley, and use conservation of string to construct a system of equations. Let T_1, T_2, and T_3 be the tension of the strings connected to m, M, and μ, and a_p be the acceleration of the movable pulley.

$$\begin{cases} a_m = T_1/m - g = -a_p \\ a_M = T_2/M - g \\ a_\mu = T_3/\mu - g \\ R(a_M - a_p) = -r(a_\mu - a_p) \end{cases} \implies R\left(\frac{T_2}{M} + \frac{T_1}{m} - 2g\right) = -r\left(\frac{T_3}{\mu} + \frac{T_1}{m} - 2g\right)$$

$$\begin{cases} \text{Above equation} \\ T_1 = T_2 + T_3 \\ T_2 r = T_3 R \end{cases} \implies \begin{cases} T_1 = 431\,\mathrm{N} \\ T_2 = 326\,\mathrm{N} \\ T_3 = 105\,\mathrm{N} \end{cases}$$