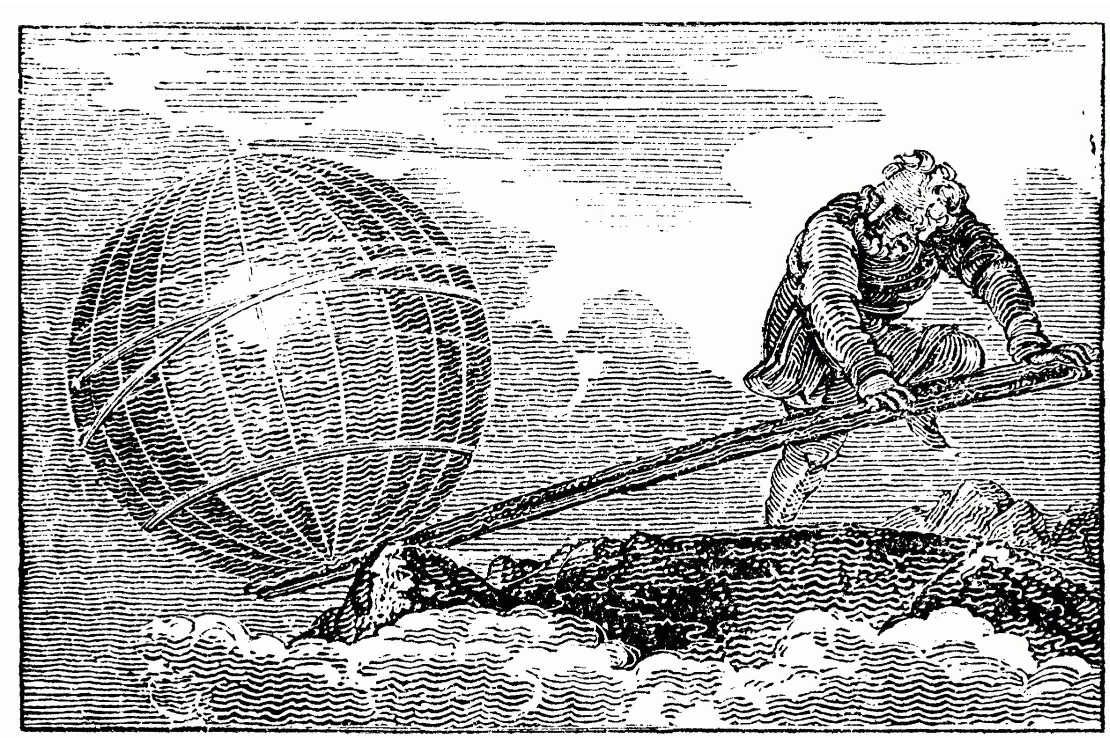
Science Olympiad
Machines C
Las Vegas Invitational

December 19, 2020



Section B Solutions

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Feedback? Test Code: *2021SOLVI-MachinesC-Shear*

Section B: Free Response

Points are shown for each question or sub-question, for a total of 90 points.

1. (14 points) April is pushing a 3000 kg box up a rough inclined plane with constant velocity. She pushes with a force of 1100 N along the inclined plane over 70 m. This process takes 7 minutes and results in a vertical displacement of 1.19 m.
   (a) (2 points) How much work is done by April, in J?
   (b) (3 points) Is the inclined plane self-locking? Explain why.
   (c) (3 points) What is the coefficient of kinetic friction between the box and the plane? (Show 5 or more significant figures)

   Once April reaches the top of the inclined plane, she finds another inclined plane on the other end, sloping at a 25° decline. This inclined plane is made from ice and has a low coefficient of kinetic friction ($\mu_k = 0.05$). She conjures a sled from the ether and slides down the icy ramp with a running start of $0.5\,\mathrm{m\,s^{-1}}$.
   (d) (3 points) How much time does it take her to slide down 100 m of ramp, in s?
   (e) (3 points) What is her velocity at the moment she travels 100 m, in $\mathrm{m\,s^{-1}}$?

Solution:

(a) Force over distance is work:

$$F \times d = W \implies 1100\,\mathrm{N} \times 70\,\mathrm{m} = 77\,000\,\mathrm{J}$$

(b) Find efficiency of the machine with potential energy gain and work done:

$$\frac{W_{out}}{W_{in}} \times 100\,\% = \eta \implies \frac{3000\,\mathrm{kg} \times 9.81\,\mathrm{m\,s^{-2}} \times 1.19\,\mathrm{m}}{77\,000\,\mathrm{J}} \times 100\,\% = 45.5\,\%$$

Since efficiency is less than 50 %, the inclined plane is self-locking.

(c) Use $\Sigma F = ma$ parallel and perpendicular to the plane surface:

$$\Sigma F_{\perp} = 0 \implies N - mg\cos\theta = 0 \implies N = mg\cos\theta$$

$$\Sigma F_{\parallel} = 0 \implies F - mg\sin\theta - mg\mu\cos\theta = 0 \implies \mu = \frac{F - mg\sin\theta}{mg\cos\theta}$$

$$\sin\theta = \frac{1.19\,\mathrm{m}}{70\,\mathrm{m}},\ \cos\theta = \frac{\sqrt{(70\,\mathrm{m})^2 - (1.19\,\mathrm{m})^2}}{70\,\mathrm{m}} \implies \mu = 0.0204$$

(d) Use a similar $\Sigma F = ma$ setup to find the constant acceleration on the sled, then do kinematics.

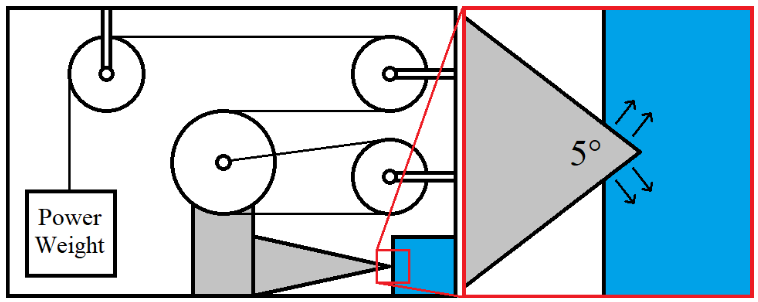
$$\Sigma F = mg\sin\theta - mg\mu\cos\theta = ma \implies a = g(\sin\theta - \mu\cos\theta) = 3.70\,\mathrm{m\,s^{-2}}$$

$$d = \frac{a}{2}t^2 + v_0 t \implies t = 7.22\,\mathrm{s}$$

(e) Set up kinematic equation and solve for v_f:

$$v_f^2 = v_0^2 + 2a\Delta x \implies v_f = 27.2\,\mathrm{m\,s^{-1}}$$

2. (29 points) Your friend devised a groundbreaking drill design that they are hoping to patent. The design, shown below, consists of a system of four pulleys and a novel drill bit shaped like a wedge. The drill bit is a triangular prism, with a 1 cm thickness (the dimension out of the page), kept aligned by a rail on the ground. The system is powered by a lifted and lowered weight. The blue square represents a piece of ore.



(a) (2 points) Find the IMA of the drill design.

(b) (12 points) Your friend came with two ways to use the device. Method one: move the drill bit so that it is just touching the ore, release the 50 kg Power Weight™, and let the drill slowly push into the ore.

i. (6 points) Let $P(d)$ be the pressure exerted by the wedge (the arrows in the right diagram) as a function of depth (initially at 0). $P(d)$ is in pascals and d is in meters. $P(d)$ can be represented in the form ad^b, find a and b.

ii. (3 points) The compressive strength of the ore is 15 MPa. How deep can the device drill, in m?

iii. (3 points) In practice, the device is only able to drill to 75 % of the predicted depth. Give a possible reason why that is the case and provide a reasonable remedy for this inefficiency.

(c) (15 points) Due to budget cuts and high tariffs, your friend can only purchase a 20 kg Power Weight™. They decide to use the other method to operate the device. Method two: pull the 100 kg drill bit back until the weight is lifted 1.5 m off the ground, release the drill bit and let the weight fall, and, right after the weight hits the ground, the drill bit hits the ore and comes to rest.

i. (3 points) What is the speed of the drill bit once the Power Weight™ hits the ground, in $\mathrm{m\,s^{-1}}$?

ii. (3 points) How much energy is lost through this process, in J?

iii. (3 points) Let's assume the drill bit comes to rest after 0.1 s. Find the average force exerted by the ore onto the drill bit, in N?

iv. (6 points) Calculate how deep the device drills until it comes to rest, in m. *(This is a challenge problem, make sure to explain your answer in depth.)*

Solution:

(a)

$$IMA = IMA_{pulley} \times IMA_{wedge} = 3 \times \frac{1}{2\tan 2.5°} = 34.4$$

(b) i. Pressure is force over area, so find force and area as a function of depth:

$$F(d) = 50\,\mathrm{kg} \times 9.81\,\mathrm{m\,s^{-2}} \times IMA,\ A(d) = 2 \times 0.01\,\mathrm{m} \times \frac{d}{\cos 2.5°}$$

$$P(d) = F(d)/A(d) = 8.42 \times 10^5\,\mathrm{N\,m^{-1}} \times d^{-1}$$

$$\therefore a = 8.42 \times 10^5\,, b = -1$$

ii. Solving $P(d) = 1.5 \times 10^7\,\mathrm{Pa}$ for d, we get $d = 5.61 \times 10^{-2}\,\mathrm{m}$

iii. Answers vary. An acceptable answer is friction (from the pulleys, the rails, the drill bit and ore) and can be remedied with lubricant (liquid [water, oil] or solid [graphite]).

(c) i. Set up a system of equations using $\Sigma F = ma$ on the two masses ($m = 20\,\mathrm{kg}$ and $M = 100\,\mathrm{kg}$) and the relative accelerations and solve for the acceleration. This is constant, so use kinematics to find the final velocity v_f. Work as follows, with tension T and height h:

$$\begin{cases} mg - T = ma_m \\ 3T = Ma_M \\ a_m = a_M \end{cases} \implies a_M = 2.10\,\mathrm{m\,s^{-2}}$$

$$v_f^2 = v_0^2 + 2a\Delta x \implies v_M = \sqrt{2a_M\frac{h}{3}} = 1.45\,\mathrm{m\,s^{-1}}$$

ii. Compare the potential energy in the Power Weight™ to the kinetic energy in the drill bit.

$$U - K = mgh - \frac{1}{2}Mv_M^2 = 189\,\mathrm{J}$$

iii. Use impulse to find average force.

$$\Delta p = \bar{F}\Delta t = Mv_M \implies \bar{F} = \frac{Mv_M}{\delta t} = 1450\,\mathrm{N}$$

iv. Answers vary. Points were awarded for proper analysis of the problem and correct reasoning. Work-energy or force/acceleration-integration based solutions are both reasonable approaches.

3. (30 points) As we cannot conduct the device testing portion of the event, you will draft up a design of a device. The device will follow the event and construction parameters and must be able to determine a mass ratio up to 10:1. However, it **must** consist of a **class 1 lever** connected to a **class 2 lever**.

(a) (8 points) Draw a labeled device diagram with dimensions.

(b) (4 points) Make an itemized list of the materials used in the design and the tools needed.

(c) (4 points) Describe the construction process of the design.

(d) (6 points) Consider two potential sources of error and explain how you will minimize their effects.

(e) (8 points) Finally, thoroughly explain the testing process for two mass ratios: 10:1 and 3:1. In your explanation, include a diagram of the mass locations and run through the appropriate calculations.

Solution:

(a) (3 points) Device consists of a class 1 and a class 2 lever.

(3 points) Diagram properly labels major features in the device.

(2 points) Dimensions for major features are included and device fits within size restrictions.

(b) (2 points) List includes materials reasonably procurable that are shown in the diagram.

(2 points) List includes tools used in the construction process.

(c) (4 points) Construction process is fully outlined.

(d) (1 point) Each source of error considered.

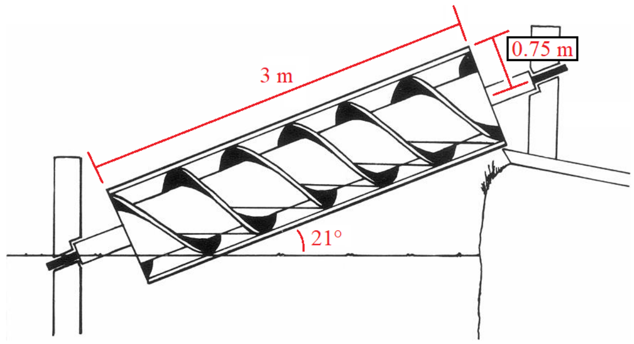
(2 points) Each explanation on how to minimize it.

(e) (1 point) Each diagram with mass locations depicted.

(3 points) Each correct calculation for the mass ratio.

4. (17 points) Shown below is a schematic of an Archimedes screw used for pumping up water. The frictionless, double-started screw makes 3 full rotations and is housed in a metal cylinder with a length of 3 m, a radius of 0.75 m, and a negligible thickness. The screw is placed at 21° with respect to the horizontal.

   The machine will be powered by a 300 W motor attached at the top of the screw. The motor's torque consists of a force applied at the radius of the screw.



(a) (2 points) What is the IMA of the machine?

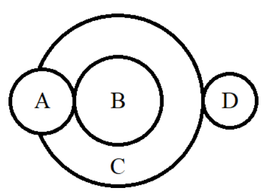
(b) (4 points) Each of the six troughs contain 20 L of water. What must the torque of the motor be to lift the water at a constant velocity, in N m? *(Hint: the density of water is* $1\,\mathrm{g\,cm^{-3}}$*)*

(c) (4 points) Compute the average flow rate of water up the screw, in $\mathrm{L\,s^{-1}}$.

(d) (7 points) After shopping around online, you find it is too expensive to buy a 300 W motor with that torque. You decide to settle with a cheaper, 200 W motor that can output a torque of 45 N m.

   i. (2 points) Looking at the value we found in (b), we can see that the required torque exceeds the motor's torque. We can design a transmission to gear down the motor. What is our target gearing ratio (x:1)? (Use 127 N m if you did not solve (b))

   ii. (5 points) The transmission will follow the layout shown below, with gears B/C axially connected and where gear A is the input (motor) and gear D is the output (screw). How many teeth (from 10 to 50) should each of the four gears have to most closely match the gearing ratio in (d.i)? *(Remember, teeth only come in whole numbers!)*



Solution:

(a)

$$IMA = IMA_{incline} \times IMA_{screw} = \frac{1}{\sin 21°} \times \frac{2\pi \times 0.75\,\mathrm{m}}{1\,\mathrm{m}} = 13.1$$

(b) Using dimensional analysis, we find $1\,\mathrm{L} = 1\,\mathrm{kg}$. Then, we know the mass of water $m = 120\,\mathrm{kg}$. Finally, we can set up the force equation:

$$mg = IMA \times \frac{\tau}{r} \implies \tau = \frac{mgr}{IMA} = \frac{120\,\mathrm{kg} \times 9.81\,\mathrm{m\,s^{-2}} \times 0.75\,\mathrm{m}}{13.1} = 67.1\,\mathrm{N\,m}$$

(c) Use conservation of power (can imagine it like energy over 1 second) to set up the equation:

$$P = Q\rho gh \implies Q = \frac{P}{\rho gh} = \frac{300\,\mathrm{W}}{1\,\mathrm{kg\,L^{-1}} \times 9.81\,\mathrm{m\,s^{-2}} \times 3\,\mathrm{m} \times \sin 21°} = 28.4\,\mathrm{L\,s^{-1}}$$

(d) i. $67.1\,\mathrm{N\,m}/45\,\mathrm{N\,m} = 1.49$. If they used $127\,\mathrm{N\,m}$: $127\,\mathrm{N\,m}/45\,\mathrm{N\,m} = 2.82$

ii. (5 points) if within 1%

(4 points) if within 5%

(3 points) if within 10%

(2 points) if within 25%

(1 point) if within 50%