Science Olympiad
Crown Point Invitational
January 25, 2025

Astronomy C Walkthrough

In this walkthrough, we will go over the free response and calculation questions. References to various sources (e.g. online material and textbooks) are included to guide the reader towards resources to learn the concepts more in depth. We hope readers find it useful.

Useful Info:

$G = 6.673 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$	$1\,\mathrm{R}_\odot = 6.957 \times 10^{8}\,\mathrm{m}$	$1\,\mathrm{R_J} = 6.991 \times 10^{7}\,\mathrm{m}$
$c = 2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$	$1\,\mathrm{M}_\odot = 1.989 \times 10^{30}\,\mathrm{kg}$	$1\,\mathrm{M_J} = 1.519 \times 10^{27}\,\mathrm{kg}$
$M_{V,\odot} = +4.7$	$1\,\mathrm{T}_\odot = 5778\,\mathrm{K}$	$\lambda_{\mathrm{H}\alpha} = 656.28\,\mathrm{nm}$

Questions 1–16: Multiple Choice [1 pt each]

1. The Sun will likely end up as a ________.
   - A. brown dwarf
   - **B. white dwarf**
   - C. neutron star
   - D. black hole
2. A nebulosity created when jets from a young star collide with gas/dust along the star's rotational axis is a ________.
   - A. Herbig Ae/Be star
   - B. T Tauri variable
   - **C. Herbig-Haro object**
   - D. Bok globule
3. A variable pre-main-sequence star with the property $M \leqslant 2\,\mathrm{M}_\odot$ contracting along the Hayashi track is a ________.
   - A. Herbig Ae/Be star
   - **B. T Tauri variable**
   - C. Herbig-Haro object
   - D. Bok globule
4. A main sequence star produces energy via ___.
   - A. gravitational contraction
   - **B. core hydrogen fusion**
   - C. shell hydrogen fusion
   - D. core helium + shell hydrogen fusion
5. At what point will a star's deuterium most likely be used as a fusion source?
   - A. During a supernova or nova event
   - B. On the asymptotic giant branch
   - C. On the main sequence
   - **D. During the protostar phase**
6. A star on the horizontal giant branch produces energy via ________.
   - A. gravitational contraction
   - B. core hydrogen fusion
   - C. shell hydrogen fusion
   - **D. core helium + shell hydrogen fusion**
7. How does a white dwarf produce energy?
   - A. Gravitational contraction
   - B. Core carbon fusion
   - C. Shell helium + shell hydrogen fusion
   - **D. White dwarfs do not produce energy**
8. Horizontal branch stars have similar luminosities, but a range of temperature. What primarily determines the temperature?
   - A. The initial mass of the main sequence star
   - **B. How much hydrogen is left surrounding the helium core**
   - C. How quickly the star ascended the red giant branch
   - D. The amount of oxygen mixed in the helium core

9. Which spectral class has the lowest surface temperature?
   - A. B
   - B. A
   - C. K
   - **D. M**

10. Which spectral class has the strongest hydrogen spectral features?
    - A. B
    - **B. A**
    - C. K
    - D. M

11. What is the primary factor that determines the strength an exoplanet transit detection?
    - A. The planet's mass
    - B. The planet's distance from the star
    - **C. The planet's diameter**
    - D. The planet's composition

12. What is the primary factor that determines the strength of an exoplanet radial velocity detection?
    - **A. The planet's gravitational force on the star**
    - B. The planet's rotational speed
    - C. The planet's ring system
    - D. The planet's surface temperature

13. Which detection method is possible for an exoplanet system seen face-on (inclination angle of 0°)?
    - A. Transit
    - B. Radial velocity
    - **C. Direct imaging**
    - D. Microlensing

14. What event marks the end of a star's ascent up the red giant branch?
    - **A. Helium flash**
    - B. Nova
    - C. Supernova
    - D. Onset of carbon fusion

15. When does a star leave the horizontal giant branch?
    - A. When shell hydrogen fusion is exhausted
    - **B. When core helium fusion is exhausted**
    - C. When the giant becomes convective
    - D. When shell hydrogen fusion begins

16. What is a brown dwarf?
    - A. A white dwarf that has cooled off enough that it doesn't give off much light anymore
    - B. A main-sequence star with a surface temperature less than $T = 3500\,\mathrm{K}$
    - C. A protostar that is cooling along the Hiyashi track
    - **D. A protostar that is not massive enough to begin core hydrogen fusion**

Questions 17–20: Identify the detection method used for each of the following exoplanets: [1 pt each]

17. WASP-17b, **Transit**
18. PSR B1257+12c, **Pulsar timing**
19. Epsilon Eridani b, **Radial velocity or Doppler spectroscopy**
20. LHS 3844b. **Transit**

Questions 21–28: Match the following statements with the corresponding object in the list below. Each choice may be used once, more than once, or not at all. [1 pt each]

A. Orion Nebula	D. LTT 9779b	G. WD 1856+534
B. 30 Doradus	E. GJ 1214b	H. Kepler-62
C. WASP-121b	F. K2-18b	I. AU Microscopii

21. This stellar remnant hosts one confirmed exoplanet, orbiting at a distance of just 0.02 au. **G**
22. Image E is an artist's concept of this ultra-hot Jupiter which exhibits potential weather patterns on its surface. **C**
23. A Neptune-like exoplanet discovered in 2020 by TESS. **D**
24. This K-type main sequence star is the host of five confirmed exoplanets. **H**
25. Gas heated to millions of degrees are highlighted in blue in the composite Image A depicting this star-forming region. **B**
26. A planetary system with two planets in this star's habitable zone. **H**
27. This exoplanet, thought to be a "water world" at its discovery, is located just 48 light-years from us. **E**
28. Image D is a cropped view of this object, which exhibits a distinctive green tint produced by the forbidden transition of doubly ionized oxygen. **A**

Questions 29–31: The planetary system about TOI-270 is extremely compact with TOI-270d orbiting only 0.07 au away.

29. [1 pt] What is the mass of TOI-270d in Earth masses?

Solution: Answers of 4.20–4.78 $M_\oplus$ were accepted.

When collecting information about the deep-sky objects make sure to consult multiple sources, especially for numerical values like mass. Wikipedia (and the journal article it takes it from) lists 4.20 $M_\oplus$, but the NASA exoplanet catalog (and a recent paper) lists 4.78 $M_\oplus$.

30. [1 pt] (T5) Name the methodology used by Hubble and JWST to study its atmosphere.

Solution: Transmission spectroscopy.

This Webb article introduces the different types of spectroscopy. A more advanced discussion of all types of exoplanet spectroscopy can be found in this article and this talk (slides).

31. [3 pts] List the three molecules whose signatures were detected by JWST.

Solution: CH_4, CO_2, and H_2O.

This information can be found on Wikipedia and this EarthSky article. Further reading of the JWST data analysis can be found in this paper and this paper.

Questions 32–34: 55 Cancri A is the host star to a large planetary system only 41 ly from the Sun. 55 Cancri e is the innermost planet in this system and its atmosphere (or the potential absence of one) is of interest to scientists. Astronomers used JWST's powerful NIRCam and MIRI to capture a thermal emission spectrum of the planet's dayside, shown in Image C, using secondary eclipse spectroscopy.

32. [1 pt] How many confirmed planets are in this system?

Solution: 5!

33. [3 pts] (T2) Describe the process of secondary eclipse spectroscopy.

Solution: Take a spectrum during secondary eclipse, where the planet passes behind the star, and subtract it from a spectrum before/after the eclipse. This leaves behind the spectrum of the planet.

34. [1 pt] What atmospheric model most closely fits JWST's collected data?

Solution: Volatile-rich.

Reading the emission spectra in Image C, we see two models: a rock-vapor atmosphere in red and a volatile-rich atmosphere in blue. Both the MIRI and NIRCam data, which are two instruments aboard JWST, more closely match the blue spectrum.

This figure can be found here. Read more about 55 Cancri e in the associated ESA article.

Questions 35–38: HD 80606b orbits in a highly eccentric orbit ($e = 0.932$, $a = 0.460\,\text{au}$, $P = 111\,\text{d}$) about its host star. Image B shows where the planet is today (Jan 25, 2025).

35. [1 pt] What type of planet is HD 80606b?

> **Solution:** A (hot) Jupiter.
>
> Learn about the different types of exoplanets from this NASA article.

36. [2 pts] (T3) Its extreme eccentricity is likely due to the gravitational influence of a second nearby star. Identify the name of this mechanism and the perturbing star.
(Hint: The perturbing star is not HD 80606)

> **Solution:** The mechanism is called the Kozai mechanism and the perturbing star is HD 80607.
>
> HD 80606, the planet's host star, is a part of a wide binary system with HD 80607. This distance star introduces a perturbatory effect on the orbit of HD 80606b, affecting its eccentricity and inclination.

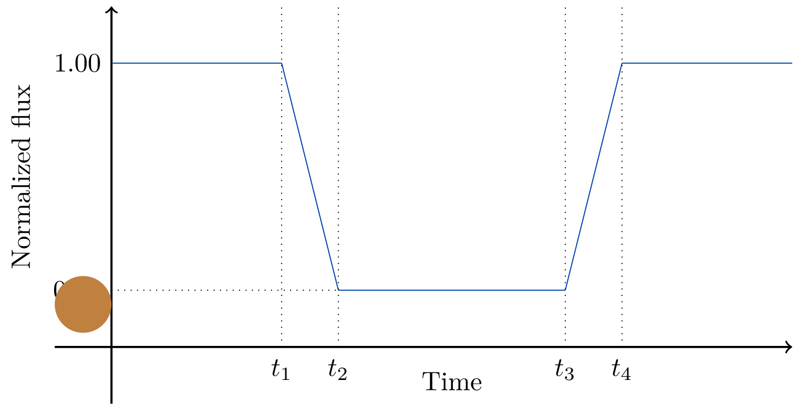
37. [3 pts] Today, you decide to use an ultra high precision spectrograph to collect a spectrum of HD 80606b. Based on the information in Image B, would you expect it to be redshifted or blueshifted? Explain why. Assume HD 80606 has zero radial velocity.

> **Solution:** From the diagram, we see that the motion of the planet has a radial component towards Earth. The Doppler effect then causes the spectrum of the planet to be blueshifted.

38. [3 pts] Today, HD 80606b is 0.815 au from its host star. How many times brighter will the host star be at periastron?

> **Solution:** Periastron is the point in an orbit where an object (a planet in this case) is the closest to the star it's orbiting about. The distance of an object at periastron from its host star is given by
>
> $$r_{\text{peri}} = a(1 - e) = 0.460\,\text{au} \times (1 - 0.932) = 0.031\,28\,\text{au}.$$
>
> Brightness of an object follows an inverse-square law. Since the planet is now $0.815/0.03128 = 26.05$ closer, it will be $(0.815/0.03128)^2 = \boxed{679}$ times brighter.
>
> *A Student's Guide to the Mathematics of Astronomy* (2013) written by Daniel Fleisch and Julia Kregenow (hereafter referred to as GMA) is a great introductory resource to many of the common mathematical relationships used in astronomy. See §2.3.2 of GMA for elliptical orbit parameters and more background on Keplerian orbits in §2.3. Also see §5.2 of GMA for discussion of brightness and the inverse-square law.

Questions 39–46: A planet is detected in circular orbit about a solar-twin (K2V) star. We observe a transit once every 32.7 d. Transits lasts about 7 hours with the precise timings given below. The star itself has an apparent visual magnitude $m = 12.2$. Assume the system is exactly edge-on (inclination angle of 90°) as viewed from Earth.



Contact	Time [hr]
t_1	0.00
t_2	1.17
t_3	5.80
t_4	6.97

39. [3 pts] How far is the planet from the star in astronomical units, au?

Solution: Exoplanets are one of the key focus areas for this year. Vox has an introduction video to exoplanets and their various detection methods. This set of questions focuses on the transit method.

As a starting point, this video lecture goes over the core ideas used in questions 39–41. This transit simulator by the folk at UNL is also a fun way to play around with the different parameters of transit light curves. Finally, *Transiting Exoplanets* (2010) by Carole A. Haswell (hereafter TE) is a wonderful introduction to transits, giving great examples and exercises alongside going over derivations.

Going back to the question, the planet first begins to cross in front of the star at t_1 and first begins to exit the star at t_3, so it takes a total of $t_3 - t_1 = 5.80\,\text{h} = 20\,880\,\text{s}$ to travel the diameter of the star. Since we know the star is a "solar-twin", it has the same diameter of the Sun, which is $D = 2\,\text{R}_\odot = 1.391 \times 10^9\,\text{m}$. We can calculate the speed of the planet to be (See §1.3.1 in GMA)

$$v_p = \frac{D}{t_3 - t_1} = 66\,620\,\text{m}\,\text{s}^{-1}.$$

Since the planet is in a circular orbit, we can use the relation

$$v = \sqrt{\frac{GM}{r}},$$

where v is the velocity of the object in the circular orbit (the planet), G is the gravitational constant, M is the mass of the central body (the star), and r is the radius of the circle. Plugging in our values

$$r = \frac{GM}{v^2} = \frac{(6.673 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}) \times (1.989 \times 10^{30}\,\text{kg})}{(66\,620\,\text{m}\,\text{s}^{-1})^2} = 2.991 \times 10^{10}\,\text{m} = \boxed{0.200\,\text{au.}}$$

This video gives a brief derivation and application of the circular orbit relation above.

40. [3 pts] What is the radius of the planet in Jupiter radii, R_J?

Solution: The planet first begins to cross in front of the star at t_1 and moves fully in front of the star at t_2, so its travels its own diameter in $t_2 - t_1 = 1.17\,\mathrm{h} = 4212\,\mathrm{s}$. Using the speed we found in the previous question, we find the radius of the planet:

$$R_p = \frac{1}{2}(t_2 - t_1)v_p = \frac{1}{2}(4212\,\mathrm{s}) \times (66\,620\,\mathrm{m\,s^{-1}}) = 1.403 \times 10^8\,\mathrm{m} = \boxed{2.01\,\mathrm{R_J}.}$$

41. [2 pts] Oh no! I spilled my coffee on the plot! What is the value of the normalized flux during transit?

Solution: The star gets dimmer by an amount proportional to the surface area of the star blocked by the planet. During the eclipse, the normalized flux drops from 1.00 to

$$f = 1 - \left(\frac{R_p}{R_\star}\right)^2 = 1 - \left(\frac{1.403 \times 10^8\,\mathrm{m}}{6.957 \times 10^8\,\mathrm{m}}\right)^2 = \boxed{0.96.}$$

Read §1.4 of TE for more information.

42. [3 pts] (T1) If I asked you to consider that the planet itself has a blackbody temperature $T_p = 1500\,\mathrm{K}$, how much of a difference, as a percentage, would that make in your previous answer?

Solution: In the previous question, we pretended the planet has a blackbody temperature $T_p = 0\,\mathrm{K}$. This meant that the answer only depended on the radius of the planet. In reality, all planets emit some residual heat (albeit much less than their host stars), which we can calculate using the Stefan–Boltzmann law. (See §3.2.2 in GMA.) We receive an incident power of $\sigma T_p^4 \pi R_p^2$ from the planet. Adding it to the power from the star and normalizing it, we get

$$f = \frac{\sigma T_\star^4(\pi R_\star^2 - \pi R_p^2) + \sigma T_p^4 \pi R_p^2}{\sigma T_\star^4 \pi R_\star^2} = \frac{R_\star^2 - R_p^2}{R_\star^2} + \left(\frac{T_p}{T_\star}\right)^4 \left(\frac{R_p}{R_\star}\right)^2.$$

So, as a fraction of our answer to the previous question, which we denote at f_o, we find

$$\frac{f}{f_o} = 1 + \frac{\left(\frac{T_p}{T_\star}\right)^4}{\left(\frac{R_\star}{R_p}\right)^2 - 1} = 1 + \frac{\left(\frac{1500\,\mathrm{K}}{5778\,\mathrm{K}}\right)^4}{\left(\frac{6.957\times10^8\,\mathrm{m}}{1.403\times10^8\,\mathrm{m}}\right)^2 - 1} = 1.00019.$$

It would increase our answer by less than $\boxed{0.02\,\%}$. Not much of a difference!

43. [2 pts] How far away is the star from Earth in parsecs, pc?

Solution: Solve for d in the distance modulus equation (See §5.3 in GMA), setting $m - M = 7.5$, to find

$$m - M = 5\log_{10}\left[\frac{d}{10\,\mathrm{pc}}\right] \quad \Longrightarrow \quad d = \boxed{316\,\mathrm{pc}.}$$

44. [2 pts] What is the maximum angular separation between the star and the planet as seen from Earth in arcseconds?

Solution: Knowing the distance to the star system and the distance between the star and planet, we can use some trigonometry on a very long, skinny triangle or we can use the small-angle formula. (See §4.2 in GMA for a discussion of angular size.) Either way, we get

$$\alpha = (206\,265'') \times \frac{0.200\,\text{au}}{316\,\text{pc}} = \boxed{6.33 \times 10^{-4}{''}.}$$

Make sure to convert units! This angle is very small, only 0.6 milliarcseconds.

45. [3 pts] Careful spectroscopic measurement of the star reveals that the wavelength of the Hα line is seen to vary by as much as 10^{-4} nm as the planet orbits. What is the mass of the planet in Jupiter masses, M_J?

Solution: This is a Doppler shift problem. (See §3.3 in GMA.) Solving for the maximum radial speed v_r, we get

$$\frac{\Delta\lambda}{\lambda_\text{true}} = \frac{v_r}{c} \quad \Longrightarrow \quad v_r = \frac{10^{-4}\,\text{nm}}{656.28\,\text{nm}} \times (2.998 \times 10^8\,\text{m}\,\text{s}^{-1}) = 45.7\,\text{m}\,\text{s}^{-1}.$$

For binary systems in circular systems, the conservation of angular momentum leads us to some nice relations between mass, radius (of the orbit), and velocity:

$$\frac{M_A}{M_B} = \frac{r_B}{r_A} = \frac{v_B}{v_A},$$

where A and B correspond to the two objects in the system. In our problem, we have $M_p v_p = M_\star v_\star$, so we compute M_p to be

$$M_p = \frac{45.7\,\text{m}\,\text{s}^{-1}}{66\,620\,\text{m}\,\text{s}^{-1}} \times (1.989 \times 10^{30}\,\text{kg}) = 1.364 \times 10^{27}\,\text{kg} = \boxed{0.898\,\text{M}_\text{J}.}$$

46. [3 pts] (T4) Would you consider this to be a habitable planet? Why or why not?

Solution: It is only 0.2 au away from its star and has a surface temperature of 1500 K. Given its mass and radius, it is likely a hot Jupiter. So it is not conducive to humans living there!