Science Olympiad
MIT Invitational
January 25, 2025

Astronomy C Walkthrough

In this walkthrough, we will go over each of the subsections in Sections B (Free Response). References to various sources (e.g. online material and textbooks) are included to guide the reader towards resources to learn the concepts more in depth. We hope readers find it useful.

Section A: Multiple Choice

This section consists of 20 multiple choice(-ish) questions about general astronomy concepts. Each question is worth 2 points, for a total of 40 points.

Stars begin their lives as collapsing dust and gas from ___1___. This material, more often than not, is rotating and leads to the formation of ___2___ which produce ___3___. ___4___ this excess angular momentum, crash into the surrounding medium, and form luminous bulbs known as ___5___. Eventually, these stars begin to fuse ___6___, reaching the main sequence.

1. A. filaments; B. starfields; C. spiral nebulae; **D. molecular clouds**
2. A. bow shocks; B. stellar winds; C. dusty envelope; **D. accretion disks**
3. A. HII regions; **B. planetesimals**; **C. absorption lines**; **D. emission lines**
4. A. Viscous boundaries consolidate
   B. Bipolar jets draw out
   C. Proplyds attenuate
   D. Density waves dissipate
5. **A. HH objects**; B. Bok globules; C. T Tauri nebulae; D. shock clouds
6. **A. hydrogen**; B. helium; C. lithium; D. iron

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7. Four brown dwarfs are found with different masses. Which mass is most likely to correspond to a brown dwarf that fuses lithium? Here, M_J denotes the mass of Jupiter.
   A. $10\,M_J$
   B. $20\,M_J$
   C. $40\,M_J$
   D. $80\,M_J$

8. Estimate to order of magnitude the pressure at the center of a brown dwarf with mass $10\,M_J$ and density 10 times that of water.
   A. 10^{10} Pa
   B. 10^{13} Pa
   C. 10^{16} Pa
   D. 10^{19} Pa

9. According to observations thus far, which of the following radii is least likely to be observed for a sub-Neptune planet? Here, $R_\oplus$ is the radius of the Earth.
   A. $1\,R_\oplus$
   B. $1.75\,R_\oplus$
   C. $2.5\,R_\oplus$
   D. $3.5\,R_\oplus$

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10. On a log-log plot, a power law (i.e. $y \propto x^\alpha$) forms a ________.
    A. straight line
    B. convex curve
    C. concave curve
    D. Depends on α

11. Which of the following orbits of a comet around a star has positive total energy? (Only considering the energy of the comet)
    A. Elliptical
    B. Circular
    C. Parabolic
    D. Hyperbolic

12. A common assumption in calculating the surface temperature of a planet is that the absorption and emission surface area of the planet are equal, $A_{\text{abs}} = A_{\text{em}}$. What if instead the planet is tidally locked to its host star, with the same side of the planet always facing the star? Calculate the ratio of the planet's surface temperature in this case to the temperature in which $A_{\text{abs}} = A_{\text{em}}$.

    A. $2^{-1/4}$

    B. $2^{-1/2}$

    C. $2^{1/2}$

    D. $2^{1/4}$

13. Where would the Balmer spectral lines be most likely to be found?

    A. HI regions only

    B. HII regions only

    C. Neither HI nor HII regions

    D. Both HI and HII regions

For the next 3 questions, match a key feature (A–D) to the model.

14. Gravitational instability model of giant planet formation. **D**

15. Core accretion model of giant planet formation. **A**

16. Widely accepted model for terrestrial planet formation. **C**

    A. The formation of a solid core through the accumulation of dust and gas

    B. The direct collapse of a giant molecular cloud into a gas giant

    C. "Pebble accretion" of colliding planetesimals, leading to a phase of oligarchic growth

    D. The fragmentation of the disk into a planetary ring system, leading to the formation of planets

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17. The transit method is more sensitive to which types of planets?

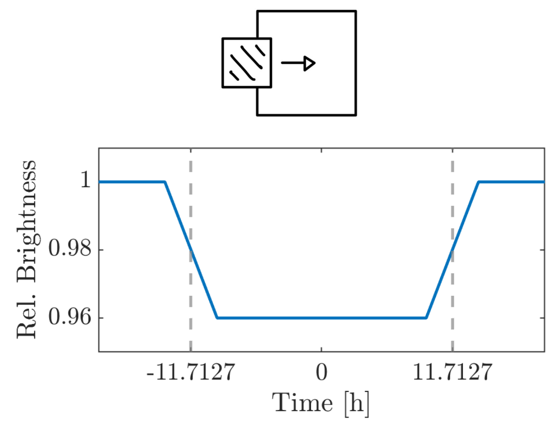
    A. Large planets with large orbits

    B. Large planets with small orbits

    C. Small planets with large orbits

    D. Small planets with small orbits

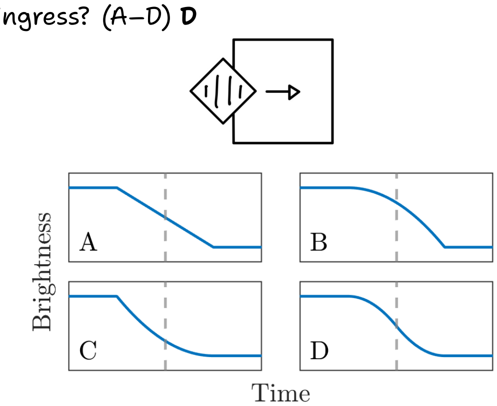
For the next 3 questions, you find yourself in an alternate reality where stars and planets are cubes. You observe a planet transiting a star and generate the light curve below. (For simplicity, we assume the star is much brighter than the planet and there is no limb darkening.) Do not show work.



18. What is the ratio between the side length of the star and the planet? (Whole number) **5**

19. You determine the planet's velocity is $1\,\text{au}\,\text{yr}^{-1}$. What is the side length of the planet in km? (1 sig. fig.) **8×10^4 km**

20. Suppose the planet is rotated 45° when it transits the star. Which of the following light curves would you expect to see during ingress? (A–D) **D**



Section B: Free Response

This section consists of 7 subsections of free response questions, each investigating one of this year's deep-sky objects. Points are shown for each question, for a total of 120 points.

Numerical answers must be provided to 3 significant figures. Please show your work: no work, no points. Partial credit may be awarded for correct work.

Subsection B-I: The Tarantula Nebula

21. [1 pt] Image 1 shows the Tarantula Nebula in which wavelength regime?
22. [3 pts] Most of the illumination in this image comes from extremely hot gas. Why is this wavelength regime a good tracer of hot gas? Explain with the radiation laws.
23. [2 pts] What is primarily responsible for the heating of this gas?

Near the center of the image, there are four bright stars in a cavity. The remaining questions will focus on this part of the Tarantula Nebula.

24. [1 pt] What is the name of this stellar association?
25. [2 pts] What are two phenomena that are primarily responsible for the cavity?
26. [1 pt] What types of stars are these?

Image 2 shows an H-R diagram of this stellar association. The probability density distribution of the stars are shown by the colored contours on the right panel; the gray contours on the left panel show the probability density distribution of Milky Way stars. Evolutionary tracks are also shown for different zero-age main sequence masses.

27. [1 pt] What evolutionary stage are the stars marked by blue circles?
28. [3 pts] The stars marked by red circles are thought to be background contaminants and not a part of the star cluster. Explain how this can be interpreted from this H-R diagram.
29. [1 pt] On each panel, there are two vertical curves marked in black. What does the left vertical curve represent?
30. [2 pts] What does the right vertical curve represent?
31. [3 pts] From both Images 1 and 2, is the star formation rate of the Tarantula Nebula significantly above, about the same, or significantly below the average star formation rate of the local group? Justify your answer with both images.

Solution: *(by April)*

The source of this image is Chandra photo album. Many DSOs have a Chandra photo album, and they usually provide good multi-wavelength interpretation of images. The answer to question 23, for example, can be found directly on this web-page.

21. X-ray.

22. Thermal X-ray emission must come from extremely high temperatures. This can be seen from Planck's law or Wien's law. From Wien's law, for example, an object which radiates predominantly in the X-ray (~1 nm) has a temperature of $\sim 10^9$ K; the hotter the object, the more it will emit in shorter wavelengths.
23. Shocks from stellar winds of OB type stars
24. Accepted answers: NGC 2070, R136, RMC136, R136a.
25. Supernovae [1 pt], stellar winds of OB type stars [1 pt]. Supernovae and stellar winds are the primary reasons why much star-forming gas is eventually expelled from a star-forming region, quenching star formation and leaving e.g. an open cluster.
26. O-type [0.5 pts] and Wolf-Rayet stars [0.5 pts]. (This is just a fact you would have to know.)

It's useful to be able to interpret H-R diagrams of clusters. In fact, H-R diagrams were first made for clusters, since we only need to plot the apparent magnitude (the distance to all the stars in the cluster are the same), and we have learned a great deal about stellar evolution from looking at the H-R diagrams of different types of star clusters.

Usually, the stars in a cluster form around the same time, so looking at the **main sequence turnoff point** should give you an estimate of the age of the system. Bright, massive stars evolve off the main sequence first, so the main sequence turns off left to right. It should always be possible to identify the main sequence of an H-R diagram; even the oldest clusters will have the lower mass stars still on the main sequence! From there, you can identify the rest of the evolutionary stages.

27. Post main-sequence (any post-main sequence evolutionary stage accepted). I made a mistake during the test-writing, and as a result there is some ambiguity for the answer to this question.
28. The red and blue dots distinguish stars which are likely past the TAMS (terminal-age main sequence) from those on the main sequence. This question was thrown out.
29. Zero-age main sequence (ZAMS).
30. Terminal age main sequence (TAMS). The main sequence that one typically sees on H-R diagrams is actually the zero-age main sequence, i.e. where stars begin when they first begin nuclear fusion. On the main sequence, stars will slowly get hotter and brighter as the hydrogen content of the core decreases, so the TAMS is not the same curve as the ZAMS.

    One common answer that I saw was the Hayashi or Henyey tracks. First of all, the Hayashi and Henyey tracks are not a single "track" like the main sequence; the pre-main sequence (PMS) track of each star depends on its mass. Additionally, the PMS phase is quite short, $\sim 10^7$ years for a sunlike star (compared with $\sim 10^9$ years for the red giant phase, and 10^{10} for main sequence), so it's quite unlikely to see so many stars in the PMS.
31. Significantly above. (This is something you should know; the Tarantula Nebula is famous for being one of, if not the, most active star-forming region in the local group!) From Image 1, we see a lot of thermal X-ray emission which must come from hot, massive stars (e.g. O-type main sequence or Wolf-Rayet) which only exist in young, actively star-forming regions. This is because the lifespans of these massive stars are quite short. From Image 2, we can see that the population of R136 is very young, since only the most massive stars have left the main sequence. Again, one estimates the age of a star cluster by looking at the main sequence turnoff point!

Subsection B-II: The Orion Nebula

32. [2 pts] Identify which image(s) depict the Orion Nebula. For each image, which telescope(s) took them?

33. [2 pts] Calculate the distance to this object in light years if its absolute magnitude is −4.1 and its apparent magnitude is 4.0.

34. [1 pt] Revise your estimate of the distance if 0.05 mag of extinction was unaccounted for in the previous calculation.

The Orion Nebula has apparent dimensions of 65 $\times$ 60 arcmin. Assume that the nebula is disk-shaped and appears as an ellipse in the night sky, with the semiminor axis independent of inclination.

35. [3 pts] For what inclination angle(s) i would its cross-section actually trace out a circle? Assume $i = 0$ corresponds to the situation where the cross-section is perpendicular to the line of sight.

36. [3 pts] Assuming the inclination angle calculated above and that stars at the outer edge of the nebula have an orbital velocity of $2\,\mathrm{km\,s^{-1}}$, calculate the mass of the Orion Nebula in solar masses. Could this velocity measurement have been easily ascertained from a radial velocity measurement?

37. [2 pts] Use your answers to the previous questions to calculate the orbital period of stars at the outer edge of the nebula in years.

38. [3 pts] Does it make sense to characterize the dynamics of stars in this nebula with Keplerian orbits around the center of the nebula? Why or why not? In what regions of this nebula might such an approximated description be valid?

Researchers discovered a eclipsing binary of brown dwarfs named 2MASS J05352184-0546085 in the Orion Nebula and found that the less massive component was actually more luminous!

39. [3 pts] Propose a possible explanation for this observation and a follow-up measurement that could be conducted to confirm this hypothesis.

Image 3 shows supersonic "bullets" that were discovered in this nebula.

40. [1 pt] What is the name of the electromagnetic spectrum in which this image taken? Be as specific as possible.

41. [1 pt] What is the composition of the blue-glowing tips of the bullets?

42. [1 pt] What is the composition of the orange trailing edges of the bullets?

Solution: *(by Sahil)*

32. In addition to Image 3 (which is mentioned later in the problem), Image 6 depicts the Orion Nebula in X-ray (using XMM-Newton) and IR (using Spitzer). You needed to get both the image number and both telescopes correct for full credit as they cover different spectral ranges.

33. This is fairly simple distance modulus and most teams obtained the correct answer, although some forgot to convert parsecs to light years.

34. If you forget where to add extinction in the distance modulus, recall that the effect of extinction is that objects now appear dimmer because of interstellar dust rather than simply being further away. Therefore, the distance estimate including extinction will be lower than the estimate neglecting it. Computing $d' = 10^{(m-M-A+5)/5} \approx 1330$ ly, where A is the extinction.

35. It's important to visualize the geometry of the problem here. Let a, b be the actual physical semimajor and semiminor axes of the ellipse and a', b' be the observed axes. From the information in the problem, We have $b' = b = a = a' \cos i$. Thus $\cos i = b'/a' = 60/65$. Some teams had $\sin i$ instead – a sanity check is that $i = 0^\circ$ should correspond to the case of seeing the perfectly circular orbit face on ($i = 90^\circ$ corresponds to seeing the orbit edge on).

36. The orbital velocity given in the problem was intended to be the actual orbital velocity. We can calculate the radius of the orbit using $a = d\theta$ where $d \approx 1330$ ly (you can use the result from either Q33 or Q34). To estimate the mass, use $F = ma$ for centripetal motion to write $v^2/a = GM/a^2 \implies M = v^2 a/G \approx 3300$ solar masses. The radial velocity measurement is not easy because the nebula is close to face on (recall that radial velocity is based on the object moving towards/away from the observer, so seeing the nebula edge on is ideal).

37. The period is given by $T = 2\pi a/v$.

38. See key.

39. Check out Stassun et al., ApJ (2007) and Reiners et al., ApJ (2007).

40. See key.

41. See key. This image was obtained using Gemini.

42. See key. The trailing edges are not ionized, so molecular hydrogen is still present.

Subsection B-III: WASP-17b

WASP-17b is a famous exoplanet that was discovered in 2009.

43. [1 pt] Identify the detection method used in its discovery.
44. [1 pt] What is significant about the orbital motion of this exoplanet?
45. [2 pts] What is the ratio of the density of this exoplanet to that of Jupiter if it has a radius twice that of Jupiter and mass half that of Jupiter?

The phenomenon in the previous part was probed using the Rossiter-McLaughlin effect.

46. [2 pts] If a star is rotating, one part of its photosphere will be moving towards the observer. What will happen to a spectral line measured from this part of the star?
47. [3 pts] As WASP-17b transits across its host star, what happens to a spectral line measured from the star? Explain what is happening physically.

Quartz clouds were detected in the atmosphere of WASP-17b in 2023 through transmission/absorption spectroscopy, as shown in Image 7.

48. [1 pt] Name the range of the electromagnetic spectrum in which this detection was performed. Be as specific as possible.
49. [3 pts] Why was the finding of pure quartz significant? How could quartz be distinguished from the more common alternative in exoplanet atmospheres in a measurement of the planet's spectrum?
50. [2 pts] Physically, what causes the resonance from quartz in the absorption spectrum at around 8.5 µm?
51. [4 pts] In addition to the quartz feature, there is a sharp resonance around 4–5 µm and a broader resonance around 7 µm. What causes each of these resonances? Propose one reason why the latter resonance is much broader than the former.

Solution: *(by Sahil)*

43. See key.

44. See key.

45. We have $\rho/\rho_J = (M/M_J)/(R/R_J)^3 = (1/2)/2^3 = 1/16 \approx 6\,$**%.**

46. This is the Doppler effect for light: relative motion that brings the source and observer closer together results in blueshifts for the observer (and further apart leads to redshifts).

47. See key for explanation.

48. Full credit only for mid-IR. It's helpful to note that mid-IR is a common spectral range for molecular spectroscopy because a lot of molecules have resonances ("fingerprints") in the mid/far-IR region. This and the following parts were based partly on Coulombe et al., *Nature* (2023).

49. See key for explanation.

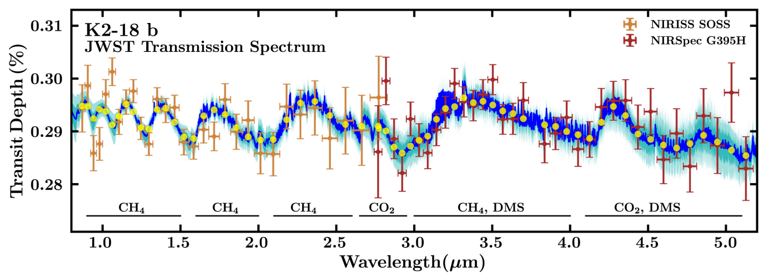
50. Excitation of vibrational modes (phonons) are responsible for the mid-IR absorption resonances.

51. See key.

Subsection B-IV: K2-18b

This subsection is the JS9 lab! Access chandra.si.edu/js9/. Here, we will use observations made by Spitzer between 3170 nm and 3950 nm to look at a transit of K2-18b.

52. [2 pts] The transmission spectrum of K2-18b is given below. Spitzer Channel 1 takes images in the 3170 nm to 3950 nm band. Do any spectral lines in the transmission spectrum of K2-18b occur in this range of wavelengths? If so, for what compound(s)?



Now, on the JS9 window, select `[File > Open remote]` and enter `bit.ly/mit25-js9-star`. Select `[Open]`, and wait for the file to load in.

This image is an image of the star K2-18 shortly before K2-18b transits in front.

53. [1 pt] On what date were these pre-transit observations taken?
54. [1 pt] If we wanted to make a second follow-up observation of a K2-18b transit as soon as possible after this one, how many days would we have to wait?
55. [2 pts] What is the observed counts of the star, in mJy? (This is the default unit of the `net_counts` column in the `[Counts in Regions]` analysis.)
56. [2 pts] There appears to be a bright spot in the bottom left corner of this image. If you can't see it, make sure `[Scale > log]` is selected. What is the angular distance (in arcseconds) to this spot?
57. [2 pts] We can say with high certainty that this spot is NOT K2-18b. What is one feature of this spot that clues us into this?

Now, on the JS9 window, select `[File > Open remote]` and enter `bit.ly/mit25-js9-transit`. Select `[Open]`, and wait for the file to load in.

This file is a subtraction of two images. We took the original image of K2-18 (star) that you just observed, and subtracted the image with K2-18b transiting in front. Thus, this file gives the brightness difference between the star, and the star with the planet transiting.

58. [3 pts] What is the value measured in the lowest-value pixel in this image (in counts)? Why might a value like this occur if the subtracted images are not perfectly aligned?
59. [2 pts] What is the counts difference between the star, and the star with K2-18b transiting in front, in mJy? *(Hint: Image subtraction does not change how* `[Counts in Regions]` *works.)*
60. [2 pts] Using these measurements, estimate the ratio of the cross-sectional area of K2-18b to the star, K2-18 (assuming that the star radiates uniformly over a perfect circle, and the planet blocks 100 % of emission over a perfect circle).
61. [4 pts] Given that the radius of K2-18 is 3.26×10^5 km, estimate the radius of K2-18b. Report:
    (a) Your radius estimate of K2-18b, in km.
    (b) The accepted value of the radius of K2-18b, in km.
    (c) The percent error of your measurement.

Solution: *(by Rio)*

WIP

Subsection B-V: LTT 9779b

LTT 9779b is a hot Neptune discovered using TESS. It has a high albedo of 0.8.

62. [1 pt] What atmospheric phenomenon on Earth (a part of the water cycle!) increases its albedo?
63. [2 pts] The dayside of LTT 9779b has a peak wavelength of 1.16 μm. What is its temperature?
64. [2 pts] Based on your answer in question 63, does the phenomenon in question 62 occur on the dayside of LTT 9779b? Why or why not?
65. [2 pts] What is the current explanation for the exoplanet's high albedo?

Traditional theories about hot Neptunes predict that their close proximity to their host star results in rapid atmospheric stripping, which is why they are so scarce.

66. [2 pts] Discuss one current theory for how the atmosphere of LTT 9779b has been able to survive.

Solution: *(by Robert)*

62. Cloud formation/condensation or ice formation/freezing. Clouds and ice are highly reflective, which increases Earth albedo. On the other hand, evaporation of water into water vapor does not increase albedo.
63. Using Wien's law, we can compute the surface temperature of LTT 9779b's dayside as

$$T = \frac{b}{\lambda} = \frac{2.898 \times 10^3\,\mu\text{m K}}{1.16\,\mu\text{m}} = \boxed{2500\,\text{K}.}$$

64. Regardless of your answer to Q62, 2500 K is well above 100 °C and 0 °C. Note that no credit was received if your answer to Q62 was incorrect.
65. The current explanation for the planet's high albedo is the formation of silicate and titanium clouds due to its extremely high metallicity (400 times the solar concentration). Read more about it in Hoyer et al., *A&A* (2023).
66. One possible theory, by the aforementioned paper, is that the planet's high albedo cooled the planet and suppressed atmospheric escape. Ferńandez et al., *MNRAS* (2023) observes LTT 9779 to be X-ray faint. If the photoevaporation of planetary atmospheres is X-ray driven, then it is likely LTT 9779b's atmosphere would survive. Jenkins et al., *Nat Astron* (2020) suggests a "late inward migration followed by [Roche-lobe overflow] may have eroded a Jupiter-mass planet down to a hot Neptune."

Subsection B-VI: PSR B1257+12

A pulsar—a rapidly spinning "corpse" of a massive star—is one of the last places you'd expect to see a planet. PSR B1257+12 is the host of not one, but three of these extreme planets. The time of arrival (TOA) of its pulses are compared to three different models to produce the TOA residuals in Image 8.

67. [2 pts] Compute the wavelength of the pulses in meters.

68. [2 pts] The top panel shows the residuals of fit to a "standard timing model without planets". How does this model model the TOAs?

69. [2 pts] The large residuals in the top panel lead astronomers to incorporate the effect of planets on the pulse TOAs. In this second model, astronomers assume the influence of each planet on the pulsar acts independently. Explain why this is a reasonable initial assumption.

70. [2 pts] The second model folds in the effect of each planet independently and its residuals are shown in the middle panel. Why are there still significant TOA residuals?

Taking the effect from the last question into account, astronomers found good agreement with the measurements as seen by the small residuals in the bottom panel.

Solution: *(by Robert)*

The plots in Image 8 were taken from and the following questions were inspired by Konacki & Wolszczan, ApJ (2003).

67. Using the "standard wave relationship", we can compute the wavelength using the frequency 430 MHz:

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8\,\mathrm{m\,s^{-1}}}{430 \times 10^6\,\mathrm{Hz}} = \boxed{0.697\,\mathrm{m}.}$$

68. The aim of Q68–70 are to step through the models used for the time of arrivals (TOAs) from top to bottom in order of increasing complexity. Each TOA corresponds to an absolute time that pulse was measured. **The simplest model of this timing corresponds to an idealized pulsar in a vacuum, rotating at a fixed rate/frequency.** And so, we receive pulses at a constant rate. For example, if we receive pulses at time 0, 3, 6, and 9 milliseconds, then we can make an educated guess that the pulsar is rotating with a period of 3 ms, or 333 Hz.

    Strobel's Astronomy Notes gives an introduction to pulsars. Chapter 6 of *Essential Radio Astronomy* by Condon and Ransom provides a more in-depth coverage of them.

    What is shown in the top panel is the errors, or residuals, when we subtract out this fixed interval between TOAs. PSR B1257+12 has a rotation period of a bit over 6 ms, so the error we see of $\sim$2 ms is a significant deviation! There must be some other effect at play...

69. In the top panel, we see the residuals (circles and triangles) are fit to a combination of multiple frequencies (solid line). Each of these frequencies are contributed by a single planet orbiting about the pulsar, which "pulls and pushes" it and consequently shortening or lengthening the interval between TOAs.

    As an aside, I saw many teams mentioning transits. Pulsar timing is not based on the planet periodically passing in front of the pulsar, but rather the planet's gravity perturbing the motion of the pulsar. If transits were observed, there would be sharp jumps in the residuals every orbit period, corresponding to multiple pulses being blocked by the planet. Doing a back of the envelope calculation, the typical pulsar has a radius of ~10 km and planets (in the Solar System) have radii of 2000–70 000 km orbiting at a speed of ~10 km s^{-1}. So a transit of PSR B1257+12 would take at least 400 s and block over 50 thousand pulses! Another important consideration is that the likelihood of being able to view the transit at all would be extremely low given how small a pulsar is!

Since pulsars have masses typically on the order of a solar mass, this would be orders of magnitude greater than the mass of potential planets. Since gravity scales linearly with mass, **the motion of each planet would dominated by the pulsar** and we can model each one with a Keplerian orbit.

But why would astronomers want to remove this effect? Because modeling the effect of each planet individually is much simpler! They can be directly added together to fit the residuals. If you consider the effect of planet B on planet A's effect on the pulsar, we end up having to solve a much, much harder (pretty much intractable) problem. (See the three-body problem.) There's no need to complicate things if a simple model would work.

Some teams mentioned that the planets are "far" from each other, focusing on the inverse-square law of gravity. This reasoning wasn't accepted. We can take a look at the Solar System and see that the Sun is over 1000 times more massive than the largest planet, Jupiter. On the other hand, consider two planets A and B (in circular orbits) about their star, with B closer to the star than A. The furthest B can be from A is 2 times the distance of A to the star. This only contributes a 4 times reduction in gravity.

70. The effect of the (individual) planets are introduced to the model and produce the middle panel residuals. It seems to work fairly well, bringing the timing errors down from 2 ms to 20 μs. However, there seem to still be some significant residuals that exhibit a pattern...

    This is due to a 3:2 mean motion resonance between planets c and d! This means that for every two revolutions of planet d about the pulsar, planet c completes three revolutions. This introduces another perturbing frequency in the system, something we ignored when we assumed the planets acted independently!

Introducing this last piece, we get residuals matching the ~2 μs measurement error of the Arecibo telescope. By iteratively increasing the complexity of a model and reanalyzing the residuals, we can see how impactful each piece is!

Subsection B-VII: WD 1856+534

WD 1856+534 is a white dwarf in a triple star system. It hosts a single giant planet, WD 1856+534b.

71. [2 pts] Sketch the orbital configuration of the system. (For illustration purposes, you may assume the triple star system, as well as the exoplanet, lie in a single orbital plane.) Label each star as well as the center of mass, and show the orbit of WD 1856+534b. The diagram should be (very) roughly to scale.

72. [1 pt] The most common spectral classification for white dwarfs is DA. What do the "D" and "A" indicate, respectively?

73. [2 pts] Hydrogen lines for WD 1856+534 are very weak and were initially undetected. What implications does this have for measuring the mass of the planet?

For questions 74–75, refer to Image 9, which shows the infrared (Spitzer 4.5 μm) flux of WD 1856b in μJy as a function of its age and mass, computed from models of brown dwarfs and giant planets.

74. [2 pts] Suppose we have a planet with mass 15 M_J and age 7 Gyr, and that its distance to Earth is twice that of WD 1856b. What is its infrared flux in μJy?

75. [3 pts] Image 10 shows the visible (Gran Telescope Canarias) and infrared (Spitzer) transit light curves of WD1856+534b. Combined with the model presented in Image 9, explain how astronomers were able to infer that WD 1856+534b is a planet (rather than e.g. a brown dwarf).

WD 1856+534b is unusual because of its tight orbit, with a semi-major axis of (0.0204 ± 0.0012) au. Recall that, for example, that the sun is expected to engulf Earth's orbit when it becomes a red giant. The question is then how WD 1856+534b either survived the engulfment of the envelope or migrated inward from a much wider orbit.

76. [2 pts] One hypothesis for explaining WD 1856+534b's orbit is called *common envelope evolution*. Describe what a common envelope is in binary evolution, and how it can tighten the orbit of the exoplanet.

77. [2 pts] Name another astrophysical system whose formation may involve common envelope evolution.

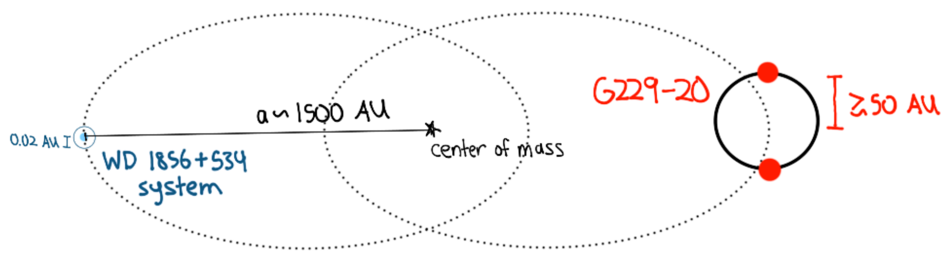
78. [2 pts] What is the energy source for the ejection of the envelope?

79. [3 pts] Several brown dwarf–white dwarf binaries with short periods are thought to have formed via common envelope evolution. Explain why this hypothesis may be less suitable for WD 1856+534b, which has a much longer period and smaller mass, with the common envelope hypothesis.

80. [2 pts] An alternative explanation involves the gravitational influence of the other stars in the triple star system. Give the name of this mechanism and briefly explain it.

Solution: *(by April)*

71. (0.5 pts for each: two ellipses, center of mass, G229-20 system, WD1856+534 system. Distance labels not necessary.)



How would we draw this system? The relevant facts you should know are that we have a white dwarf (WD1856+534) and a close binary (G229-20) orbiting each other, and that the masses of these two objects are comparable. That is all the information you need to construct a plausible orbit of this system!

To recap how general 2-body orbits go: Kepler's 1st law says that planets orbit in ellipses with the star at the focus. In general binary system, each object orbits in an ellipse with the center of mass at a focus. In the case of a planetary system, the center of mass is just inside the star, since the star is much more massive than the planets; that's why we see the planet moving in an ellipse with the star at the focus. But in this case, the masses are comparable: the combined mass of G229-20 is $\sim 0.7\,M_\odot$ and WD1856+534 has mass $\sim 0.6\,M_\odot$, so we draw two ellipses, one for G229-20 and one for WD1856+534, and have them share a focus.

Why can we treat this as a 2-body system, when there are clearly at least 3 large bodies (and a planet)? Well, G229-20 is a tight binary: the separation between G229-20 is much smaller than the separation between G229-20 and WD1856+534. That lets us approximate G229-20 as one "object" and the WD1856+534 system as another object.

72. White dwarf [0.5 pts], hydrogen lines present [0.5 pts]. Some put the temperature of A-type stars for "A". While it's true that *main-sequence* A-type stars tend to have temperatures around 10 000 K, white dwarfs have a different spectral classificiation system than main sequence stars.

    Main sequence A-type stars are also characterized by strong Balmer lines. (That's why they're called A-type stars in the first place!) For main sequence stars, it turns out that it's the temperature of the star that determines the strength of the Hydrogen lines. But since white dwarfs are the former cores of stars, the spectral classification of the white dwarf actually tells us about its composition. That's why most white dwarfs are DA!

73. The only way we can measure the mass of a planet directly is through the radial velocity method, which measures the velocity of the star caused by the gravitational pull of the planet on the sun. We do this by looking at how the spectral lines of the host star shift and using the Doppler effect. Weak or undetectable Balmer lines means that it'll be difficult to observe this effect.

Image 9 was taken from this paper. To make this plot, astronomers used models of gas giant and brown dwarf atmospheres to predict a planet's infrared luminosity as a function of mass and age, and used the distance to WD 1856 to translate this into expected measured infrared flux. Notice how for a given mass, the flux of a planet decreases with system age. This makes sense because giant planets cool and contract over time.

74. From the plot, we can read off 30 μJy. μJy is a unit of flux, i.e. power per unit area. That is, it's a unit of measured or apparent brightness. This plot was calibrated for WD 1856b. The inverse squared law tells us that the flux is inversely proportional to the square of the distance, so doubling the distance will decrease the flux by a factor of 4. Therefore, we would measure an infrared flux of 30 / 4 = 7.5 μJy.

75. **The optical and infrared transit depths are the same, which means there is no detectable infrared emission from the transiting object**. Why? A relatively cool object, such as a planet or brown dwarf, emits more in the infrared than visible. We can check this using Wien's law: an object that emits primarily at 700 nm (red, and longer wavelengths than this are infrared) has a temperature of around 3000 K, and anything cooler than this has its peak wavelength in the infrared (or even longer wavelengths). Since we expect a planet or brown dwarf to be much cooler than a star (red dwarfs are around 3000 K, since they emit primarily red light), it's safe to assume that its infrared luminosity is much greater than its visible luminosity. Therefore, the transit depths being the same means that we don't detect anything at all from the planet.

    Using the model presented in the graph, one can then set a bound on the mass of the object. This is actually what the grayed out region shows: Spitzer's sensitivity limit is around 15.5 μJy, so the planet can't be in the gray region. This lets us set an upper bound on the mass of the planet. The horizontal line that separates brown dwarfs from planets is the 13 M_J threshold for deuterium fusion.

    Note that while a very old, ~13 M_J brown dwarf is compatible with the lack of detectable infrared emission (see red sliver on the right), intuition should tell us that the system should not be so old. The universe itself is only around 14 Gyr old, after all! And since white dwarfs are dead stars, old white dwarfs cool over time and become undetectable, but clearly WD1856 is still quite bright!

An aside on infrared and black body radiation: I saw many competitors put something along the lines of "infrared is associated with heat", "hot objects emit infrared", or even "infrared is heat". This is a misconception probably caused by e.g. heat goggles or heat sensors are just infrared sensors. But heat goggles work for detecting human-temperature warm objects, which by astronomical standards is not hot at all! Using Wien's law again (or examining some black body radiation curves), we can see that humans are too cool to emit much in the visible, but emit thermal radiation in the infrared. So there is nothing special about infrared that makes it associated with heat. All objects with a temperature will emit some black body thermal radiation: it's just that at the temperatures we're used to dealing with on Earth, this radiation falls in the infrared.

76. Common envelope evolution occurs when the **envelope of the primary extends past its Roche lobe, causing mass transfer and runaway shrinking of the orbit** until the secondary (planet) is engulfed inside the star. The orbit is then tightened further via drag forces inside the star. This loss of orbital energy (consisting of both kinetic and gravitational potential energy) goes into expelling the common envelope, which ends the common envelope phase.

77. Examples of accepted answers: AM CVns, X-ray binaries, binary black hole mergers/compact binary coalescences.

    Common envelope theory is a big area of research in binary evolution, especially because it's among the leading hypotheses for how tight binaries form, such as the merging black holes that LIGO detects form! (LIGO is a gravitational wave detector, and also the only way we can detect binary black holes.) An internet search will provide a good visual of how common envelope evolution works. Although there's a lot of support for the common envelope phase in simulations, we have yet to have a confirmed observation of a binary in a common envelope.

78. Orbital energy (see above)

79. In order to have had a common envelope, the planet needs to have contributed enough orbital energy to expel the envelope. More massive objects can contribute more orbital energy (orbital energy is proportional to mass), and if the resulting orbit is very tight, then it's evidence that the orbit shrunk and lost a lot of gravitational potential energy to the envelope. So, WD 1856+534b's smaller mass and relatively wider orbit means it would not have lost enough orbital energy to eject the envelope and end a potential common envelope phase.

    However, if there were other planets that were engulfed first that helped contribute to the expelling of the envelope, then a common envelope phase is still possible!

80. The Lidov-Kozai mechanism occurs when a binary orbit is perturbed by a third object (here, the red dwarf binary G229-20) on an outer orbit.

    Common envelope and Lidov-Kozai mechanisms are the leading hypotheses for how tightly orbiting binary systems form in general. For example, it's also a hypothesis for how LIGO merging black holes form! In a dense star cluster such as a globular cluster, it's possible for many three-body encounters to produce inward migration of a binary. this diagram from this paper shows both the common envelope and Lidov-Kozai mechanisms for producing these systems.

    The punchline is—many different areas of astronomy and physics are all connected!