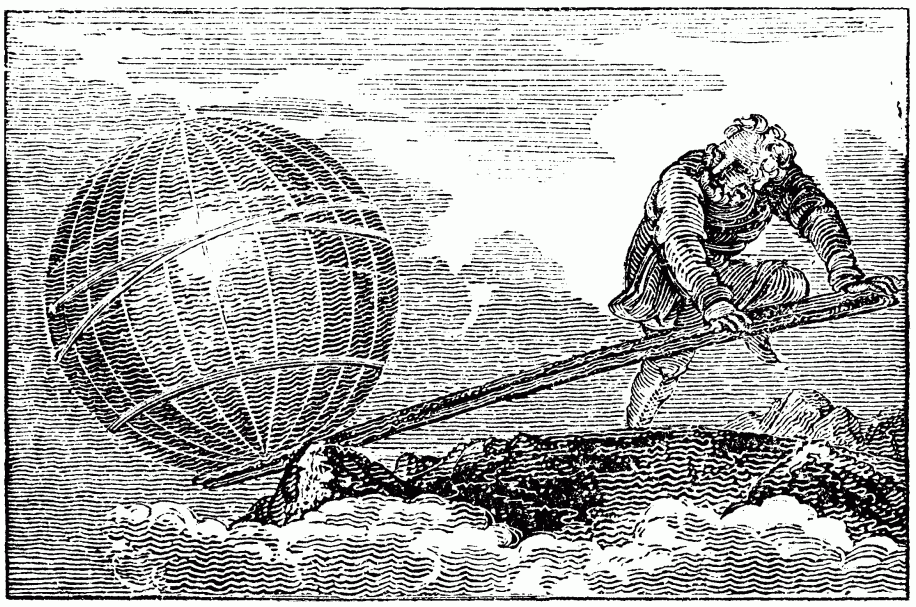
Science Olympiad
Golden Gate Invitational
January 31, 2026

Machines C



Directions:

- Each team will be given a total of **50 minutes** to complete the exam and device testing.
- There are **two sections**: **Section A** (Machinery) and **Section B** (Mechanery).
- **Do not write on the exam**. Only write on your answer sheet.
- For calculation questions, **work will be graded**. Please show all your work.
- Unless otherwise specified, report numerical answers to **three significant figures**.
- Whenever needed, take the acceleration of gravity g to be $9.81\,\mathrm{m/s^2}$.
- Tiebreakers, in order: §A, 10, 8, 13, 4.
- After the tournament, the exam will be available online at `robertyl.com/scioly`
- Best of luck!

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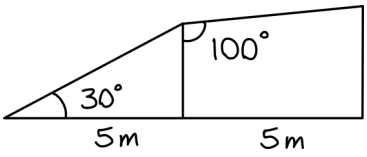
Section A: Machinery

This section generally focuses on "classic" questions on simple and compound machines. There are six questions, for a total of 42 points.

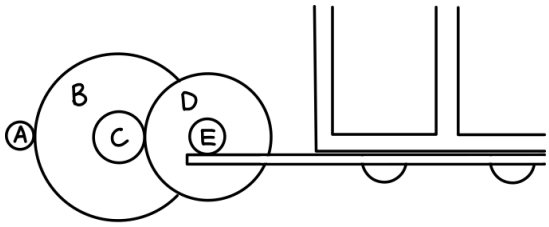
1. Rayleigh has a compound inclined plane.



   (a) [3 pts] What is its (energy) average IMA?

   (b) [7 pts] Rayleigh rolls a tube of aluminum foil ($\rho_{\text{Al}} = 2.7\,\text{g/cm}^3$) without slipping with a force of 300 N through the center of mass of the tube and parallel to the surface of the plane. The tube has an inner and outer diameter of 20 cm and 30 cm and is 50 cm long. Assume the transfer between the inclined planes is negligible. How long does it take to reach the top, in seconds?

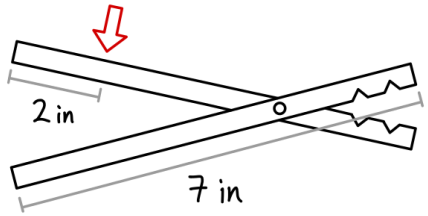
2. The entrance to a gated community is controlled by a 300 kg, 18 ft wide gate that slides on 20 small wheels, each with a rolling friction of 0.1. The gate mechanism is controlled by a motor that drives gear A, which is connected to a rack and pinion. The diameters of the gears A, B, C, D, and E are 5 in, 35 in, 8 in, 20 in, and 4 in, respectively.



   (a) [2 pts] Compute the time-averaged power, in watts, needed to close the gate in 5 seconds.

   (b) [3 pts] Evaluate the minimum torque from the motor, in N m, to open the gate back up.

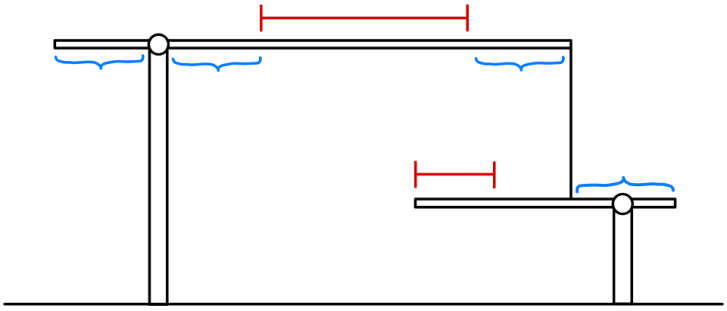
3. A wire stripper is shown below with the input force represented by an arrow. The rivet and the first stripping location are 1.5 in and 0.5 in away from the right end of the stripper. The stripper has a wedge angle of 25°.



   (a) [1 pt] What class lever are the arms of the wire stripper?

   (b) [4 pts] What is the IMA of the wire stripper?

4. Phoebe's back! Recall that she possesses a (pretty much) massless, frictionless, indestructible lever arm that is 1 m long. Fortunately, this year she was able to find another lever arm with the same properties that is 50 cm long. Preparing for the event, she builds her device which is depicted below where the two levers are connected by a lightweight string. The fulcrums are 90 cm apart. Masses may only be affixed to the levers in the red ranges. The upper and lower red ranges have lengths 40 cm and 15 cm. The curly brackets all indicate the same length, with the rightmost one centered on the lower fulcrum.

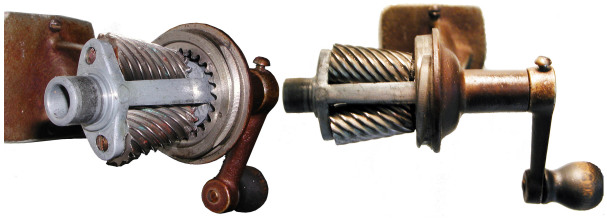


   (a) [2 pts] In one sentence, level with Phoebe and explain to her what's wrong with her device.

   (b) [3 pts] Taking your advice, she fixes a pulley above the levers at the horizontal position of the string and wraps the string over it. Assume the size of the pulley is much smaller than the lever. What is the greatest mass ratio she can determine?

   (c) [5 pts] Her measurements have an absolute error of ± 1 mm. For a mass ratio of 5, where should she place the two masses to minimize the maximum mass ratio percent error? Justify your answer.

5. A manual pencil sharpener is shown below and can be described as a planetary gearset with a **fixed** ring gear, two planet gears, and a missing sun gear. The ring gear has 24 teeth and each planet gear has 10 teeth. The ring gear has a diameter of 3 cm and the handle arm is 8 cm long.



   (a) [2 pts] How many teeth would the sun gear have if it was included?

   (b) [3 pts] Rotating the handle at an angular rate of 2 rotations per second, what is the angular rate of the hypothetical sun gear in the same units? Does it rotate in the same or opposite direction?

   (c) [5 pts] A pencil (7 mm diameter) is pushed into the sharpener with 10 N of force. The helical cutters are angled 5° from the horizontal and meet to form a perfect point. If a force of 20 N is required to rotate the crank, compute the effective coefficient of friction of sharpening the pencil.

6. [2 pts] Order the following four pile types from lowest to largest angle of repose: gravel, wet sand, round dry sand, and angular dry sand.

Section B: Mechanery

This section covers more typical topics in classical mechanics. There are eight questions, for a total of 58 points.

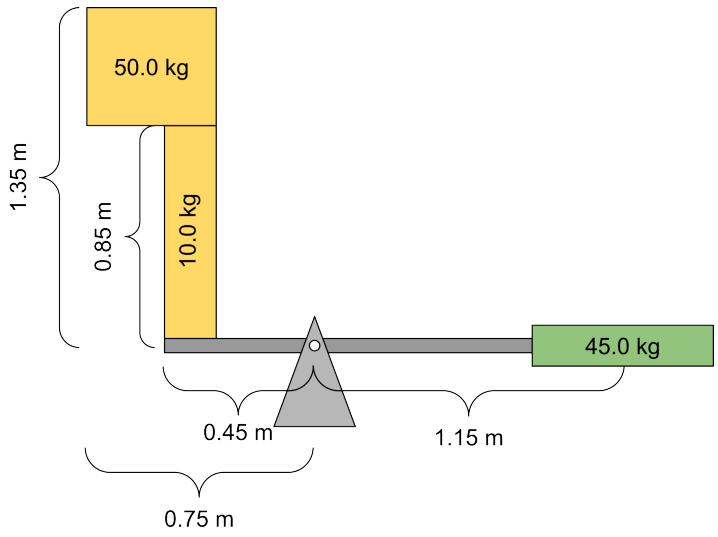
7. Imagine an inclined plane with four objects at the top: a hollow cylinder, a disk, a solid ball, and a hollow ball. Each object has the same mass and radius. They are released from rest and all of them roll down the slope at the same time.

   (a) [2 pts] Order the objects from slowest to fastest down the inclined plane if they roll without slipping.

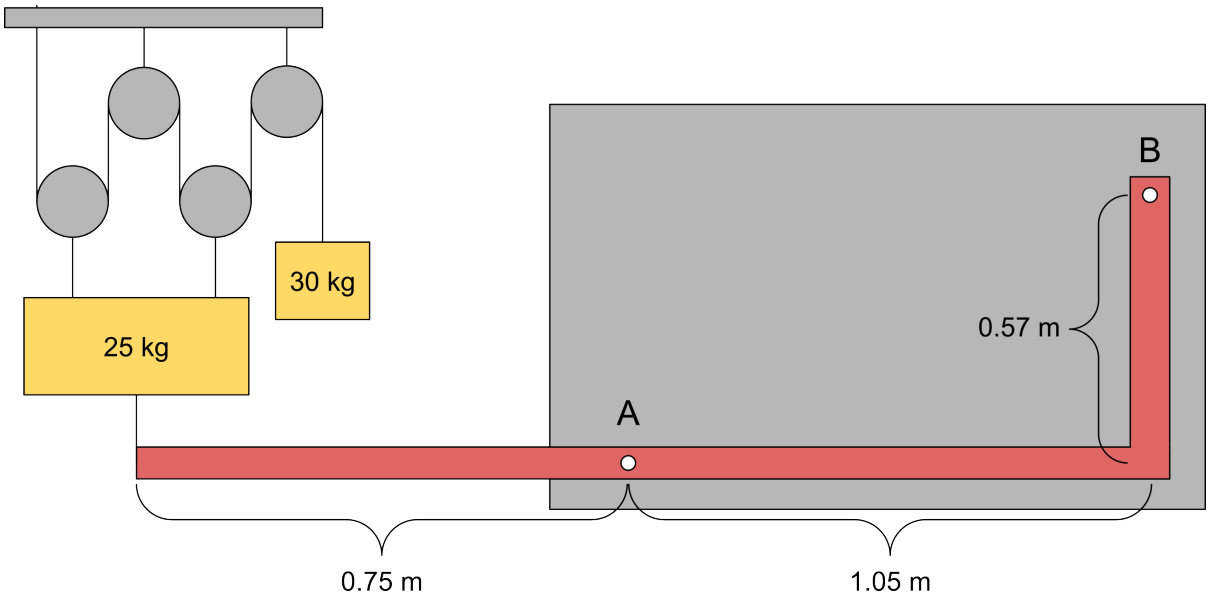
   (b) [2 pts] Order the objects from slowest to fastest down the inclined plane if there is no friction.

   (c) [5 pts] Return to the case where each object rolls without slipping. Measuring the angular momentum of each object as soon as it reaches the bottom of the inclined plane, we find the object with the **greatest angular momentum** has $1/\sqrt{2}\,\mathrm{kg\,m^2/s}$. Based on this information, identify the angular momentum of all four objects. Express all quantities in the form $a/\sqrt{b}\,\mathrm{kg\,m^2/s}$ where a and b are coprime positive integers.

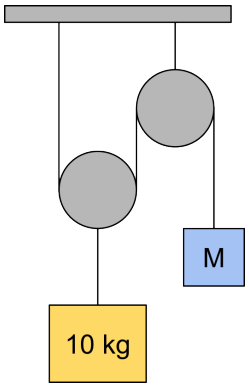
8. [8 pts] The diagram below depicts a thin, massless lever with a 45.0 kg block on the right and a **square** 50.0 kg block attached to a rectangular block of 10.0 kg mass on the left. Assuming these blocks are rigidly attached (no sliding), determine the angle (in degrees), with respect to the horizontal, of the lever at equilibrium. *Note that the figure is not drawn to scale.*



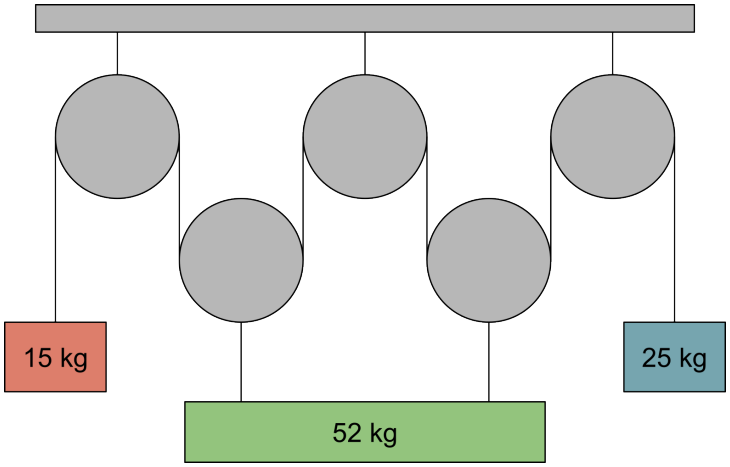
9. [6 pts] In the diagram below, a massless red "L" shape structure is attached to a strong, rigid plate via two points: A and B. Determine the reaction forces, in newtons, at points A and B if you know the reaction at B in the horizontal direction is 150 N to the left. *Leave your answer in vector component form.*



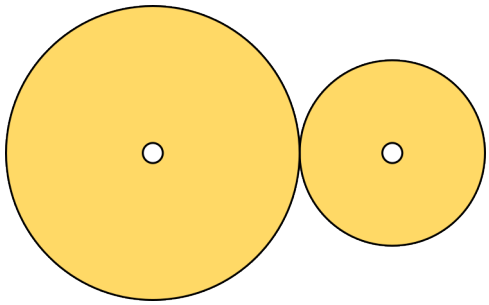
10. [4 pts] In the diagram below, what should the mass of M be for the 10 kg block to experience an acceleration of 2.7 m/s^2 downwards?



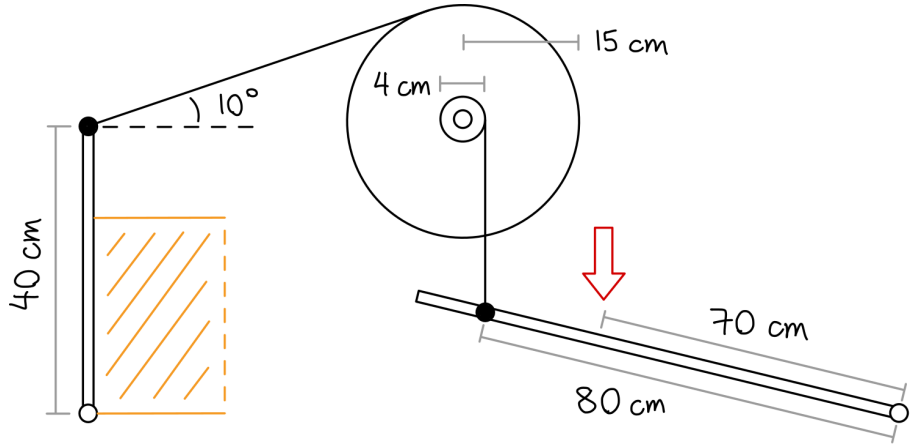
11. [8 pts] Using the diagram below, find the acceleration of the 25 kg block, in m/s^2. Specify a coordinate system (i.e., down is negative and up is positive).



12. [5 pts] Two wheels are in direct contact with one another and each one spins about its own fixed axle. One wheel has a radius of 0.478 m and the other has a radius of 0.952 m. Both wheels are solid, have a uniform density of 23.5 kg/m^3, and have a thickness of 3.57 cm. If the coefficient of friction between the two wheels is 0.423 and the stationary normal force experienced at the contact point is 52.7 N, what is the maximum angular acceleration, in rad/s^2, the small wheel can undergo without causing slip in the contact between the wheels?



13. The black squirrels on campus have a conundrum. They've hidden a cache of acorns in a tree hollow but are worried they'll get stolen. To close it up, they've installed a (uniform and rigid) **square** door that is held closed with a drawbridge-like system. The acorns can be modeled as small (relative to the size of the door) uniform spheres with a density of $1\,\mathrm{g/cm^3}$. The bulk of randomly arranged acorns has a packing density of 63.5 %. Finally, the acorns can be modeled as a fluid, so the pressure they apply on the door linearly increases with depth: $P = \rho g h$.

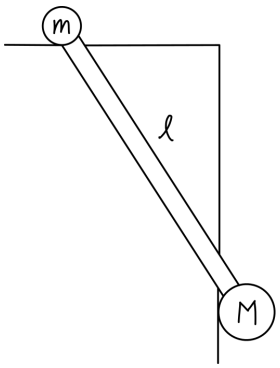


The orange, shaded region represents the cache of acorns. The small white circles are pivot points and the small black circles are points where ropes are tied.

(a) [5 pts] To maintain security, the squirrels will take shifts standing where the red arrow is to keep an eye on the cache. Their presence also acts as a counterweight (of 1.5 lb) to keep the hollow shut. What's the maximum height of acorns they can store, in cm?

(b) [3 pts] In a few months, spring will have sprung. Little do they know, the acorns were stolen! (So there is no internal acorn pressure.) Subject only to gravity, the trapdoor falls open from upright at rest. How fast is it rotating right before it hits the ground? **Express your answer in terms of g and ℓ** (the height of the door, which is 40 cm).

14. Fun fact: The founder of Trader Joe's, Joe Coulombe, is a Stanford double grad (BA and MBA)! After getting your week's groceries (and snacks) from the Town & Country TJ's, you hang your canvas TJ's tote on the edge of a wooden seat rest, which can be modeled by a taut, massless rope of length ℓ over a square edge, connected to masses m and M which both have a coefficient of friction μ with the wood. The rope does not experience friction.



(a) [4 pts] First, we'll assume $m \ll 1$ and $M \gg 1$. What is the maximum distance mass m can be from the edge while the tote remains at rest? Express your answer in terms of ℓ and μ.

(b) [6 pts] Now $m = M \gg 1$ as well. Let $\mu = 0.35$ and the rope be at a 60° angle such that m is further from the edge than M. Find the accelerations of m and M **in terms of g** when released from rest (i.e., m and M accelerate upwards at $0.789g$ and to the left at $219g$).