Science Olympiad
Golden Gate Invitational
January 31, 2026

Machines C Walkthrough

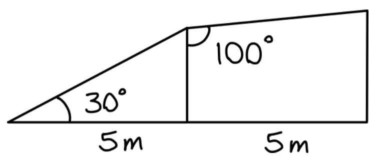
In this walkthrough, we will go over select questions in more depth than the key.
We hope readers find it useful.

Question 6–12 (except 7(c)) were written by Nir.
Questions 1–5, 7(c), 13, 14 were written by Robert.

Section A: Machinery

This section generally focuses on "classic" questions on simple and compound machines. There are six questions, for a total of 42 points.

1. Rayleigh has a compound inclined plane.



(a) [3 pts] What is its (energy) average IMA?

(b) [7 pts] Rayleigh rolls a tube of aluminum foil ($\rho_{\mathrm{Al}} = 2.7\,\mathrm{g/cm^3}$) without slipping with a force of 300 N through the center of mass of the tube and parallel to the surface of the plane. The tube has an inner and outer diameter of 20 cm and 30 cm and is 50 cm long. Assume the transfer between the inclined planes is negligible. How long does it take to reach the top, in seconds?

Solution:

(a) IMA can be computed by dividing the effort distance by the resistance distance. The choice of 5 m is irrelevant to the answer and can be generalized to some distance d. The height traveled by a box on the incline is $d(\tan 30° + \tan 10°)$ and distance traveled is $d(\sec 30° + \sec 10°)$. Taking their ratio, we find

$$\frac{d(\sec 30° + \sec 10°)}{d(\tan 30° + \tan 10°)} = \boxed{2.88.}$$

Partial credit was awarded for taking the arithmetic average of the IMA (3.88).

(b) First, we compute the mass of the tube to be

$$M = w \times \frac{\pi}{4}(d_o^2 - d_i^2) \times \rho = 53.0\,\mathrm{kg}$$

and its moment of inertia as

$$I = w \times \frac{\pi}{64}(d_o^4 - d_i^4) \times \rho = 0.431\,\mathrm{kg\,m^2}.$$

With this, we can compute the angular acceleration as

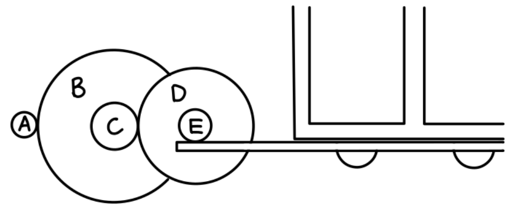
$$(300\,\mathrm{N} - Mg\sin\theta) \times 0.15\,\mathrm{m} = I\alpha \quad \implies \quad \begin{cases} \alpha = 13.9\,\mathrm{rad/s^2} & \text{for } \theta = 30°, \\ \alpha = 73.0\,\mathrm{rad/s^2} & \text{for } \theta = 10°. \end{cases}$$

We can relate this to the linear acceleration with $a = \alpha \times 0.15\,\mathrm{m}$ and with some kinematics, we get the time required:

$$t_1 = 2.35\,\mathrm{s},\ t_2 = 0.614\,\mathrm{s} \quad \implies \quad t_1 + t_2 = \boxed{2.97\,\mathrm{s.}}$$

This question was particularly involved for for one placed at the beginning of the exam.

2. The entrance to a gated community is controlled by a 300 kg, 18 ft wide gate that slides on 20 small wheels, each with a rolling friction of 0.1. The gate mechanism is controlled by a motor that drives gear A, which is connected to a rack and pinion. The diameters of the gears A, B, C, D, and E are 5 in, 35 in, 8 in, 20 in, and 4 in, respectively.



   (a) [2 pts] Compute the time-averaged power, in watts, needed to close the gate in 5 seconds.

   (b) [3 pts] Evaluate the minimum torque from the motor, in N m, to open the gate back up.

Solution:

(a) Frictional energy loss can be computed by taking the friction force over distance:

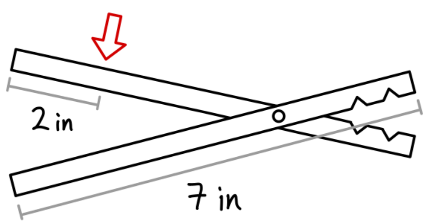
$$F_f \times d = 300\,\text{kg} \times 9.81\,\text{m/s}^2 \times 0.1 \times 18\,\text{ft} = 1610\,\text{J}.$$

Averaging this over 5 seconds, we get $\boxed{323\,\text{W.}}$

(b) The torque τ must be sufficiently large to overcome the friction. The IMA of the gear train is 17.5. The output force is

$$\frac{\tau}{2\,\text{in}} \times \text{IMA} = 300\,\text{kg} \times 9.81\,\text{m/s}^2 \times 0.1 \qquad \Longrightarrow \qquad \tau = \boxed{0.854\,\text{N m.}}$$

3. A wire stripper is shown below with the input force represented by an arrow. The rivet and the first stripping location are 1.5 in and 0.5 in away from the right end of the stripper. The stripper has a wedge angle of 25°.



   (a) [1 pt] What class lever are the arms of the wire stripper?

   (b) [4 pts] What is the IMA of the wire stripper?

Solution:

(a) Class 1, since the input and output forces are on the opposite sides of the fulcrum.

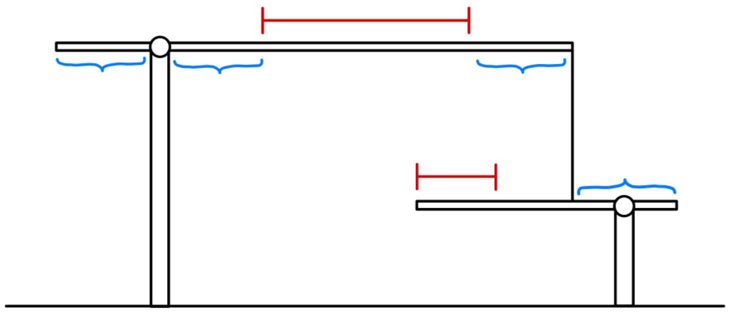
(b) Using the provided lengths, we can find

$$\text{IMA}_{\text{lever}} = 3.5\,\text{in}/1\,\text{in} = 3.5 \quad \text{and} \quad \text{IMA}_{\text{wedge}} = (2\tan(25°/2))^{-1} = 2.26.$$

Taking their combined effect, we get IMA = $\boxed{7.91.}$

Partial credit is awarded if a wedge IMA of $\cot(25°) = 2.14$ is used or the output force is considered as being doubled.

4. Phoebe's back! Recall that she possesses a (pretty much) massless, frictionless, indestructible lever arm that is 1 m long. Fortunately, this year she was able to find another lever arm with the same properties that is 50 cm long. Preparing for the event, she builds her device which is depicted below where the two levers are connected by a lightweight string. The fulcrums are 90 cm apart. Masses may only be affixed to the levers in the red ranges. The upper and lower red ranges have lengths 40 cm and 15 cm. The curly brackets all indicate the same length, with the rightmost one centered on the lower fulcrum.



   (a) [2 pts] In one sentence, level with Phoebe and explain to her what's wrong with her device.
   (b) [3 pts] Taking your advice, she fixes a pulley above the levers at the horizontal position of the string and wraps the string over it. Assume the size of the pulley is much smaller than the lever. What is the greatest mass ratio she can determine?
   (c) [5 pts] Her measurements have an absolute error of ± 1 mm. For a mass ratio of 5, where should she place the two masses to minimize the maximum mass ratio percent error? Justify your answer.

Solution:

(a) The tension on the upper lever arms does not counteract the weight of the mass.

(b) Using the lengths provided, we can deduce the blue curly brackets measure 20 cm. From this, the IMA of the upper lever is 4/3 to 4 and the IMA of the lower lever is 5/2 to 4. So the greatest mass ratio is $\boxed{16.}$

(c) If you place the lighter mass at the rightmost point of the lower range. Then the heavier mass will be at the midpoint of the upper range. The maximum percent error of the measured mass ratio will then be

$$\frac{25 \pm 0.1}{10} \times \frac{80}{40 \pm 0.1} \quad \Longrightarrow \quad 0.652\,\%.$$

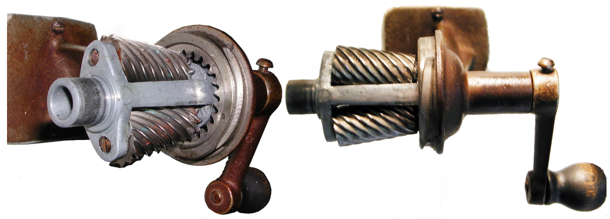
If you place the heavier mass at the rightmost point of the upper range. Then the lighter mass will be 2.5 cm from the end of the lower arm. The maximum percent error of the measured mass ratio will then be

$$\frac{37.5 \pm 0.1}{10} \times \frac{80}{60 \pm 0.1} \quad \Longrightarrow \quad 0.434\,\%.$$

So we see that the latter set of positions minimizes the maximum mass ratio percent error to be $\boxed{0.434\,\%.}$ Unfortunately for you competitors living in the real world, there are many more compounding sources of error.

It can be shown that any intermediate positions for the masses result in a higher error than the one we found. This is not necessary for full credit. The educated guess that the optimal location is at an extrema is sufficient.

5. A manual pencil sharpener is shown below and can be described as a planetary gearset with a **fixed** ring gear, two planet gears, and a missing sun gear. The ring gear has 24 teeth and each planet gear has 10 teeth. The ring gear has a diameter of 3 cm and the handle arm is 8 cm long.



   (a) [2 pts] How many teeth would the sun gear have if it was included?

   (b) [3 pts] Rotating the handle at an angular rate of 2 rotations per second, what is the angular rate of the hypothetical sun gear in the same units? Does it rotate in the same or opposite direction?

   (c) [5 pts] A pencil (7 mm diameter) is pushed into the sharpener with 10 N of force. The helical cutters are angled 5° from the horizontal and meet to form a perfect point. If a force of 20 N is required to rotate the crank, compute the effective coefficient of friction of sharpening the pencil.

Solution:

(a) Let the number of teeth of the ring, planet, and sun gears be N_r, N_p, and N_s, respectively. They are governed by the relation:

$$N_r = 2N_p + N_s \qquad \Longrightarrow \qquad N_s = \boxed{4 \text{ teeth.}}$$

(b) For a fixed ring gear, the angular rate of the carrier ω_c relates to the angular rate of the sun gear ω_s by

$$\frac{\omega_s}{\omega_c} = 1 + \frac{N_r}{N_s} \qquad \Longrightarrow \qquad \omega_s = \boxed{14\,\text{rps.}}$$

It rotates in the $\boxed{\text{same direction.}}$

(c) Since the effect of the contact forces involved are symmetric between the two helical cutters, we can consider one effective cutter. The total normal force on the surface of the pencil is $N = 10\,\text{N}/\sin 5° = 115\,\text{N}$. On the other hand, a total force of $F = 20\,\text{N} \times (8\,\text{cm}/0.875\,\text{cm}) = 183\,\text{N}$ acts through the center of the planet gears. A torque balance about the contact between the ring and planet gear gives

$$N\mu \times 1.325\,\text{cm} = F \times 0.625\,\text{cm} \qquad \Longrightarrow \qquad \mu = \boxed{0.752.}$$

If the friction location is incorrectly placed at the inner edge of the planet gear, a value of $\mu = 0.797$ is worth partial credit.

6. [2 pts] Order the following four pile types from lowest to largest angle of repose: gravel, wet sand, round dry sand, and angular dry sand.

Solution: Round dry sand particles roll easily past each other. Therefore, they have the lowest angle. Angular dry sand has more edges that can interlock between the particles. However, the particles are still small, so this would go next in the hierarchy. Gravel has the second largest angle of repose due to the particles being much larger than sand. Finally, wet sand has added cohesion from water which allows the angle of repose to be much larger.

Section B: Mechanery

This section includes more standard topics in classical mechanics. There are eight questions, for a total of 58 points.

7. Imagine an inclined plane with four objects at the top: a hollow cylinder, a disk, a solid ball, and a hollow ball. Each object has the same mass and radius. They are released from rest and all of them roll down the slope at the same time.

   (a) [2 pts] Order the objects from slowest to fastest down the inclined plane if they roll without slipping.

   (b) [2 pts] Order the objects from slowest to fastest down the inclined plane if there is no friction.

   (c) [5 pts] Return to the case where each object rolls without slipping. Measuring the angular momentum of each object as soon as it reaches the bottom of the inclined plane, we find the object with the **greatest angular momentum** has $1/\sqrt{2}\,\mathrm{kg\,m^2/s}$. Based on this information, identify the angular momentum of all four objects. Express all quantities in the form $a/\sqrt{b}\,\mathrm{kg\,m^2/s}$ where a and b are coprime positive integers.

Solution: Let M and R be the shared mass and radius of the objects. It is helpful to know that the moments of inertia for the objects can be written in the form αMR^2 where $\alpha = 1$ for the hollow cylinder, $\alpha = 1/2$ for the disk, $\alpha = 2/5$ for the solid ball, and $\alpha = 2/3$ for the hollow ball.

(a) If α of an object is greater, more gravitational potential energy must be allocated to its rotation, thereby slowing it down. So, the slowest object has the greatest α and vice versa. This gives the ordering: hollow cylinder, hollow ball, disk, and solid ball.

(b) This is a trick question. Since there is slipping, each object accelerates at the same rate and they all reach the bottom simultaneously.

(c) Let h be the height of the inclined plane and ω be the angular velocity of the object. Then, from conservation of energy, we get

$$Mgh = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}\alpha MR^2\omega^2 \quad \Longrightarrow \quad \omega^2 \propto \frac{1}{1+\alpha}.$$

Using the definition of angular momentum $L = I\omega$, we see

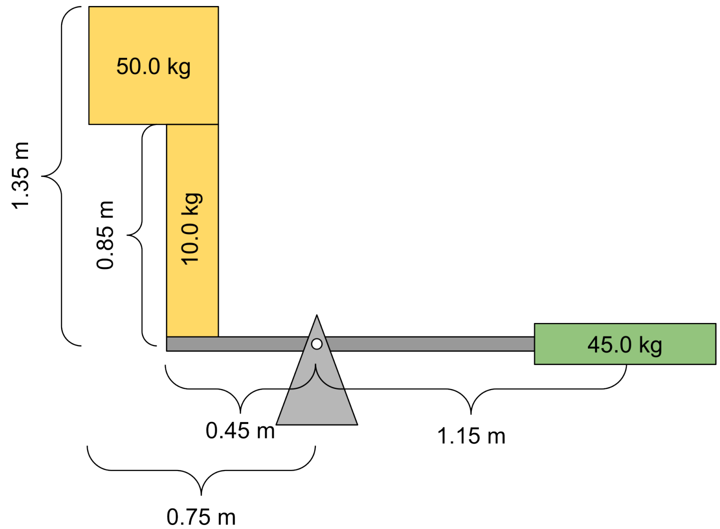
$$L \propto \frac{\alpha}{\sqrt{1+\alpha}}.$$

If we plug in our values for α, we find that the ratios for L between the objects are

$$\alpha = 1 : \frac{2}{3} : \frac{1}{2} : \frac{2}{5} \quad \Longrightarrow \quad L = \boxed{\frac{1}{\sqrt{2}} : \frac{2}{\sqrt{15}} : \frac{1}{\sqrt{6}} : \frac{2}{\sqrt{35}}}$$

which give the values for a and b we are looking for.

8. [8 pts] The diagram below depicts a thin, massless lever with a 45.0 kg block on the right and a **square** 50.0 kg block attached to a rectangular block of 10.0 kg mass on the left. Assuming these blocks are rigidly attached (no sliding), determine the angle (in degrees), with respect to the horizontal, of the lever at equilibrium. *Note that the figure is not drawn to scale.*



Solution: First, we must solve for the general dimensions of the system before we can set up the free body diagram. It is given that the 50 kg block is a square, so we know it is 0.5 m on each side. We can now solve for the locations of the centers of mass of the yellow objects:

The 10 kg object's center of mass is located 0.35 m from the fulcrum and 0.425 m above the lever.

The 50 kg object's center of mass is located 0.5 m from the fulcrum and 1.1 m above the lever.

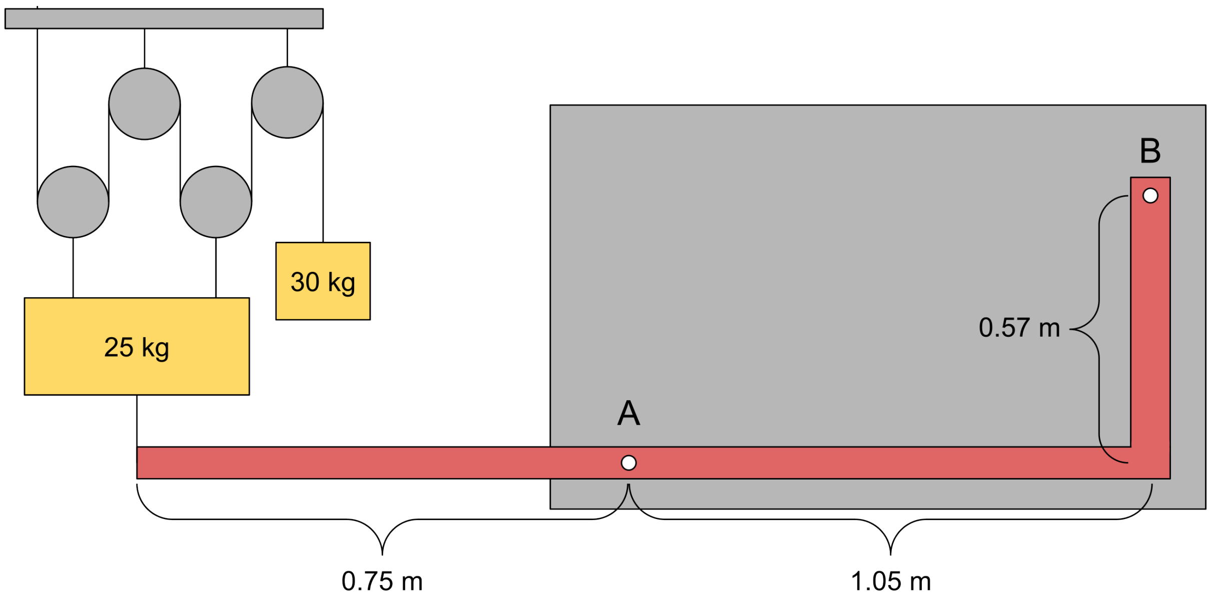
Doing a trivial sum of moments check on the system, it is clear that the moment produced by the 45 kg block outweighs the moments produced by the yellow blocks. Therefore, the lever naturally accelerates clockwise. This means the system must be rotated the opposite direction (ccw) to find the equilibrium.

The angle at which the system is in equilibrium can be found with a sum of moments equation. (This is where the heights of the yellow blocks come into play.)

$$\begin{aligned}\sum M = 0 = (45)(9.81)(1.15\cos\theta) - ((10)(9.81)(0.35\cos\theta + 0.425\sin\theta) \\ + (50)(9.81)(0.35\cos\theta + 0.425\sin\theta + 0.15\cos\theta + 0.675\sin\theta))\end{aligned} \implies \theta = 21.4^\circ$$

The system must be $\boxed{21.4^\circ \text{ counterclockwise}}$ from the horizontal.

9. [6 pts] In the diagram below, a massless red "L" shape structure is attached to a strong, rigid plate via two points: A and B. Determine the reaction forces, in newtons, at points A and B if you know the reaction at B in the horizontal direction is 150 N to the left. *Leave your answer in vector component form.*



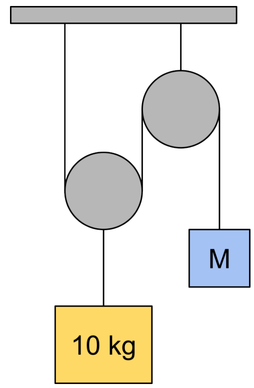
Solution: Simplifying the pulley system gives us the net force acting on the left-most edge of the "L" shape structure to be:

$$F = (4)(30)(9.81) - (25)(9.81) = 932\,\text{N}.$$

We are given force in the horizontal direction at point B, so we only have 3 forces left to find: A_x, A_y, and B_y. To do this, we can set up and solve a system of equations:

$$\begin{aligned} \sum F_x &= 0 = A_x + 150, \\ \sum F_y &= 0 = 932 + A_y + B_y, \\ \sum M_A &= 0 = (932)(0.75) - 1.05B_y - (0.57)(150), \end{aligned} \quad \Longrightarrow \quad \boxed{\begin{aligned} A_x &= -150\,\text{N}, \\ A_y &= -1516\,\text{N}, \\ B_y &= 584\,\text{N}. \end{aligned}}$$

10. [4 pts] In the diagram below, what should the mass of M be for the 10 kg block to experience an acceleration of $2.7\,\mathrm{m/s^2}$ downwards?



Solution: The 10 kg block is supported by two segments of the rope. Therefore, if the block accelerates downward with acceleration a, the mass M accelerates upward with magnitude

$$a_M = 2a.$$

Thus,

$$a_M = 2(2.7) = 5.4 \text{ m/s}^2.$$

Let T be the tension in the rope. For the 10 kg block (downward positive):

$$10g - 2T = 10a.$$

Solving for T,

$$2T = 10(g-a) \quad \Longrightarrow \quad T = \frac{10(g-a)}{2}.$$

For mass M (upward positive):

$$T - Mg = Ma_M.$$

Substitute $a_M = 2a$:

$$T = M(g+2a).$$

Equate the two expressions for T:

$$\frac{10(g-a)}{2} = M(g+2a).$$

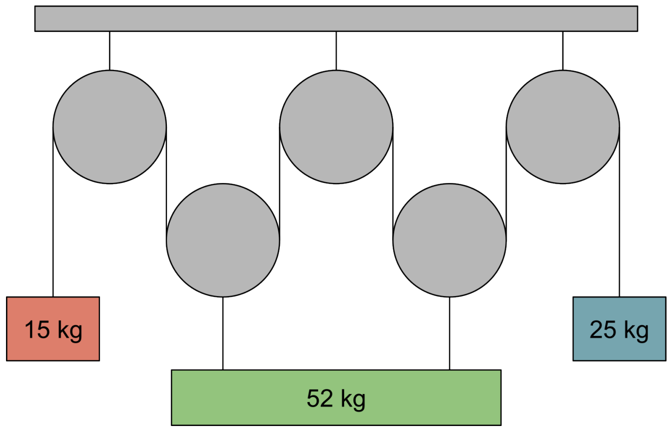
Solving for M,

$$M = \frac{10(g-a)}{2(g+2a)}.$$

Using $g = 9.8\,\mathrm{m/s^2}$ and $a = 2.7\,\mathrm{m/s^2}$:

$$M = \frac{10(9.8-2.7)}{2(9.8+5.4)} = \frac{71}{30.4} \approx \boxed{2.34\,\mathrm{kg}.}$$

11. [8 pts] Using the diagram below, find the acceleration of the 25 kg block, in m/s^2. Specify a coordinate system (i.e., down is negative and up is positive).



Solution: A Newtonian approach (i.e., using $F = ma$) can be applied to solve this problem, but it will be tedious. We detail a more direct solution leveraging Lagrangian mechanics. For notational simplicity, let $m_1 \coloneqq 15\,\mathrm{kg}$, $m_2 \coloneqq 52\,\mathrm{kg}$, and $m_3 \coloneqq 25\,\mathrm{kg}$.

First, we note that the system has two degrees of freedom: the vertical coordinate of m_1 and the vertical coordinate of the m_3. Let these be x and y respectively and, by convention, orient the positive direction upwards. Then, the constraints imposed by the strings allow us to determine the position (and thereby the rate) of the last block. Namely, the rate of change of the vertical position of m_3 is $-(\dot{x} + \dot{y})/4$.

With this, we can write the system kinetic energy T and potential energy V as:

$$\begin{aligned}
T &= \frac{1}{2}\left[m_1\dot{x}^2 + \frac{1}{16}m_2(-\dot{x} - \dot{y})^2 + m_3\dot{y}^2\right] \\
&= \left(\frac{16m_1 + m_2}{32}\right)\dot{x}^2 + \frac{1}{16}m_2\dot{x}\dot{y} + \left(\frac{m_2 + 16m_3}{32}\right)\dot{y}^2, \\
V &= m_1 g x - \frac{1}{4}m_2 g(x + y) + m_3 g y \\
&= \left(\frac{4m_1 - m_2}{4}\right)gx + \left(\frac{-m_2 + 4m_3}{4}\right)gy.
\end{aligned}$$

Applying Euler–Lagrange, we produce the equations of motion:

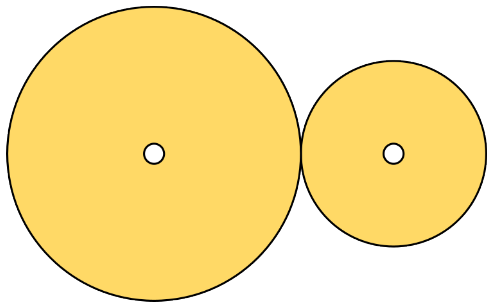
$$\begin{aligned}
\frac{d}{dt}\frac{\partial T}{\partial \dot{x}} + \frac{\partial V}{\partial x} &= \left(m_1 + \frac{1}{16}m_2\right)\ddot{x} + \frac{1}{16}m_2\ddot{y} + \left(m_1 - \frac{1}{4}m_2\right)g = 0, \\
\frac{d}{dt}\frac{\partial T}{\partial \dot{y}} + \frac{\partial V}{\partial y} &= \left(\frac{1}{16}m_2 + m_3\right)\ddot{y} + \frac{1}{16}m_2\ddot{x} + \left(-\frac{1}{4}m_2 + m_3\right)g = 0.
\end{aligned}$$

Substituting in numerical values for m_i and g and solving the system of two equations, we recover

$$\ddot{x} = -0.340\,\mathrm{m/s^2} \quad \text{and} \quad \ddot{y} = \boxed{-4.12\,\mathrm{m/s^2}.}$$

The acceleration of the 52 kg block is $1.12\,\mathrm{m/s^2}$.

12. [5 pts] Two wheels are in direct contact with one another and each one spins about its own fixed axle. One wheel has a radius of 0.478 m and the other has a radius of 0.952 m. Both wheels are solid, have a uniform density of 23.5 kg/m^3, and have a thickness of 3.57 cm. If the coefficient of friction between the two wheels is 0.423 and the stationary normal force experienced at the contact point is 52.7 N, what is the maximum angular acceleration, in rad/s^2, the small wheel can undergo without causing slip in the contact between the wheels?



Solution: The mass of the large wheel is

$$M_L = \rho \pi r_L^2 t = 23.5 \times \pi \times (0.952)^2 \times 0.0357 \approx 2.389\,\text{kg}.$$

For a solid disk, the moment of inertia is

$$I_L = \frac{1}{2} M_L r_L^2 \approx 1.0825\,\text{kg}\,\text{m}^2.$$

The large wheel is accelerated only by the friction force F at the contact point. Its rotational equation of motion is

$$\tau_L = I_L \alpha_L \quad \Longrightarrow \quad F r_L = I_L \alpha_L.$$

To avoid slipping, the tangential accelerations at the contact point must match:

$$\alpha_s r_s = \alpha_L r_L \quad \Longrightarrow \quad \alpha_L = \alpha_s \frac{r_s}{r_L}.$$

Substitute the constraint into the torque equation:

$$F r_L = I_L \left(\alpha_s \frac{r_s}{r_L} \right) \quad \Longrightarrow \quad F = \frac{I_L \alpha_s r_s}{r_L^2}.$$

Note that the inertia of the small wheel does not appear here—$\cancel{F r_s = I_s \alpha_s}$—as the small wheel is externally driven.

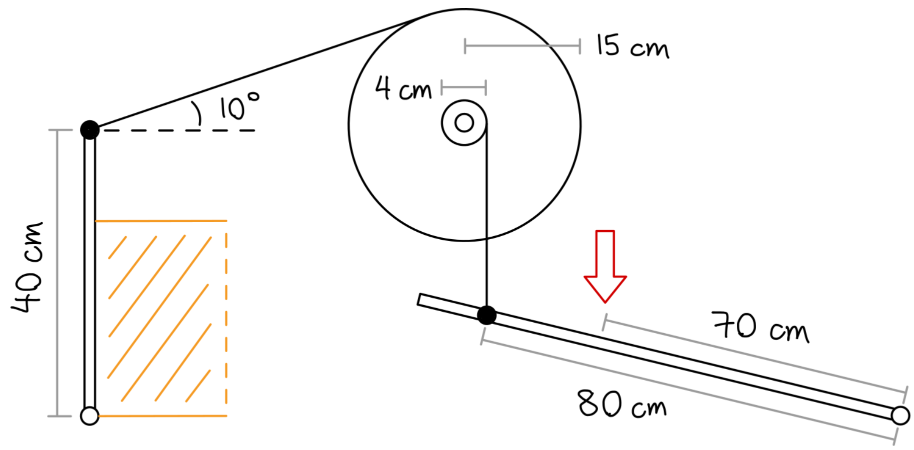
The maximum static friction force is $F_{\max} = \mu N$; so at the threshold of slipping,

$$\mu N = \frac{I_L \alpha_{s,\max} r_s}{r_L^2}.$$

Solving for the maximum angular acceleration of the small wheel,

$$\alpha_{s,\max} = \frac{\mu N r_L^2}{I_L r_s}. = \frac{(0.423)(52.7)(0.952)^2}{(1.0825)(0.478)} \approx \boxed{39.0\,\text{rad/s}^2.}$$

13. The black squirrels on campus have a conundrum. They've hidden a cache of acorns in a tree hollow but are worried they'll get stolen. To close it up, they've installed a (uniform and rigid) **square** door that is held closed with a drawbridge-like system. The acorns can be modeled as small (relative to the size of the door) uniform spheres with a density of $1\,\mathrm{g/cm^3}$. The bulk of randomly arranged acorns has a packing density of 63.5 %. Finally, the acorns can be modeled as a fluid, so the pressure they apply on the door linearly increases with depth: $P = \rho g h$.



The orange, shaded region represents the cache of acorns. The small white circles are pivot points and the small black circles are points where ropes are tied.

(a) [5 pts] To maintain security, the squirrels will take shifts standing where the red arrow is to keep an eye on the cache. Their presence also acts as a counterweight (of 1.5 lb) to keep the hollow shut. What's the maximum height of acorns they can store, in cm?

(b) [3 pts] In a few months, spring will have sprung. Little do they know, the acorns were stolen! (So there is no internal acorn pressure.) Subject only to gravity, the trapdoor falls open from upright at rest. How fast is it rotating right before it hits the ground? **Express your answer in terms of g and ℓ** (the height of the door, which is 40 cm).

Solution:

(a) The torque on the door from the weight of the squirrel is

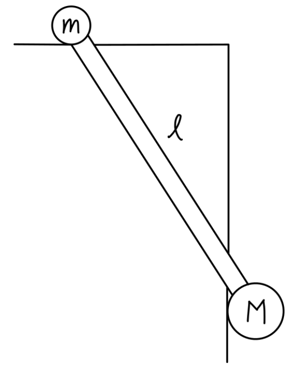
$$2.70\,\mathrm{N\,m} \times \frac{70}{80} \times \frac{15}{2} \times \cos 10^\circ = 17.3\,\mathrm{N\,m}.$$

Let h be the height of the acorns in meters. The bulk density of the acorns is $635\,\mathrm{kg/m^3}$. Then, the equivalent force of the (distributed) pressure is $1245.87\,\mathrm{N/m^2} \times h^2$ applied at a distance of $h/3$ from the ground. Setting these equal, we get $h = \boxed{34.6\,\mathrm{cm.}}$

(b) We use conservation of energy. The gravitational potential energy of a rod of length l and mass m is $mgl/2$. Let $\dot{\theta}$ be the angular velocity at the ground. Then the angular kinetic energy is

$$\frac{1}{3} m l^2 \dot{\theta}^2 = \frac{1}{2} m g l \quad \Longrightarrow \quad \dot{\theta} = \boxed{\sqrt{\frac{3}{2}\frac{g}{l}}.}$$

14. Fun fact: The founder of Trader Joe's, Joe Coulombe, is a Stanford double grad (BA and MBA)! After getting your week's groceries (and snacks) from the Town & Country TJ's, you hang your canvas TJ's tote on the edge of a wooden seat rest, which can be modeled by a taut, massless rope of length ℓ over a square edge, connected to masses m and M which both have a coefficient of friction μ with the wood. The rope does not experience friction.



(a) [4 pts] First, we'll assume $m \ll 1$ and $M \gg 1$. What is the maximum distance mass m can be from the edge while the tote remains at rest? Express your answer in terms of ℓ and μ.

(b) [6 pts] Now $m = M \gg 1$ as well. Let $\mu = 0.35$ and the rope be at a $60°$ angle such that m is further from the edge than M. Find the accelerations of m and M **in terms of g** when released from rest (i.e., m and M accelerate upwards at $0.789g$ and to the left at $219g$).

Solution: For this problem, we'll define the coordinates x and y in the usual fashion, with the origin at the ledge. Note the size of m and M are negligible.

(a) Let T be the tension in the rope and θ be the acute angle in the figure (near M). Since the system is at equilibrium and $M \gg m$, we operate under the assumption $T \gg mg$. Then, the horizontal forces on m can be written out as

$$ma_x = T\sin\theta - \mu T\cos\theta,\ a_x = 0 \quad \Longrightarrow \quad T\sin\theta < \mu T\cos\theta \quad \Longrightarrow \quad \tan\theta < \mu.$$

So, regardless of the tension in the rope. m remains stationary so long as $\theta < \tan^{-1}\mu$.

Since the choice of T is arbitrary, it can rise to an arbitrary amount such that M remains at equilibrium as well. Mentioning this fact is sufficient, but a free-body-diagram analysis of M is also accepted.

Finally, we use some trigonometry to find the maximum distance of m from the edge to be

$$\tan\theta = \mu \quad \Longrightarrow \quad d_{\text{edge}} = \boxed{\ell\frac{\mu}{\sqrt{1+\mu^2}}}.$$

(b) This problem setup is reminiscent of a classic problem that considers the dynamics of a ladder sliding down a frictionless corner. A Lagrangian approach is used there, but the introduction of friction leaves a Newtonian approach to be better suited.

As in part (a), we set up the horizontal and vertical forces on m and M, respectively, based on the unknown tension (T), the horizontal acceleration of m ($\ddot{x}$), and the vertical acceleration of M ($\ddot{y}$):

$$T\sin\theta - (mg + T\cos\theta)\mu = m\ddot{x} \quad \Longrightarrow \quad \frac{T}{m}(\sin\theta - \mu\cos\theta) = g\mu + \ddot{x},$$
$$T\cos\theta + \mu T\sin\theta - Mg = M\ddot{y} \quad \Longrightarrow \quad \frac{T}{M}(\cos\theta + \mu\sin\theta) = g + \ddot{y}.$$

Note that we only have two equations for three unknowns. The final relation comes from the constraint of the rope. Since the length of the rope is preserved, application of the time derivative gives

$$x^2 + y^2 = \ell^2 \implies x\dot{x} + y\dot{y} = 0 \implies \cancelto{0}{\dot{x}^2 + \dot{y}^2} + x\ddot{x} + y\ddot{y} = 0.$$

Since $x/y = \sqrt{3}$, we find $\ddot{y} = -\sqrt{3}\ddot{x}$.

With this and the fact that $m = M$, we solve for the unknowns:

$$\boxed{\ddot{x} = 0.205g,} \quad \boxed{\ddot{y} = -0.355g,} \quad \text{and} \quad \frac{T}{m} = 0.803g.$$