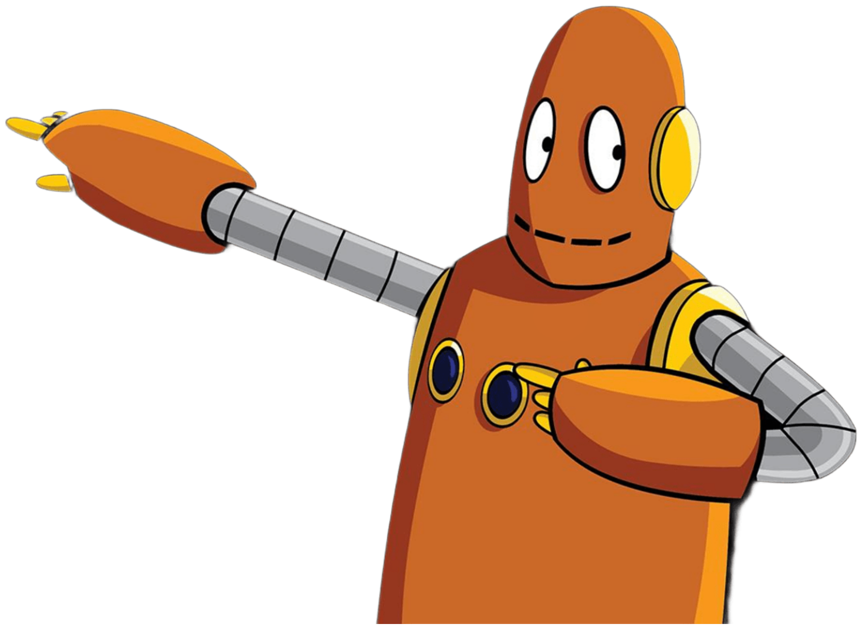
Science Olympiad
MIT Invitational
January 24, 2026

Machines C



Directions:

- Each team will be given a total of **50 minutes** to complete the exam and device testing.
- There are **two sections**: **Section A** (Multiple Choice) and **Section B** (Free Response).
- **Do not write on the exam**. Only write on your answer sheet.
- For calculation questions, **work will be graded**. Please show all your work.
- Unless otherwise specified, report numerical answers to **three significant figures**.
- Whenever needed, take the acceleration of gravity g to be $9.81\,\mathrm{m/s^2}$.
- Tiebreakers, in order: B1, B2, B5, B3, B4.
- After the tournament, the exam will be available online at `robertyl.com/scioly`
- Best of luck!

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Section A: Multiple Choice

There are 35 questions in this section, each worth 2 points, for a total of 70 points.

1. Mingjia links a three-gear train with teeth ratio 25 : 30 : 36 with another three-gear train with teeth ratio 20 : 10 : 5. Find the maximum possible IMA of this new six-gear train.
   A. 1.80
   B. 3.60
   C. 5.00
   D. 7.20

2. Jay slowly pours a mystery sand onto a circular dish forming a cone with height 3 cm and diameter 8 cm. What is the angle of repose of this sand?
   A. 22°
   B. 37°
   C. 53°
   D. 68°

3. Take a balanced seesaw of mass m_1 and length 12 m with pivot at $x = 8$ m. Jiamu (mass m_1) is on the longer end of the see-saw. Kevin (mass m_2) is on the shorter end. Which is true?
   A. $2m_1 = m_2$
   B. $m_1 = 2m_2$
   C. $5m_1 = 2m_2$
   D. $2m_1 = 5m_2$

4. Josie is working on a mousetrap vehicle for her physics class. She needs the vehicle to travel 30 m. In ideal conditions, what length of string must she wound around the vehicle's axles of diameter 3 cm given she has four wheels of radius 12 cm?
   A. 2.50 m
   B. 3.75 m
   C. 7.50 m
   D. 10.0 m

5. Meng pulls a 41 kg block up an inclined plane with incline $\theta = 23°$ and length 1.32×10^2 m. Find the work done by friction if the coefficient of sliding friction is 0.28.
   A. -1.37×10^4 J
   B. 2.74×10^4 J
   C. 1.37×10^4 J
   D. -2.74×10^4 J

Questions 6–7 refer to Parth "Pushing P" Mhaske pushing up a box of mass 9 kg on an inclined plane as shown below. The ramp has the property that for every 1 m in vertical rise, there is a horizontal displacement of 5 m.

6. What is the IMA of the ramp?
   A. 5.00
   B. 5.05
   C. 5.10
   D. 5.15

7. He pushes the box with force F_A such that it begins accelerating at 6.7 m/s^2. Find F_A.
   A. 72.5 N
   B. 77.6 N
   C. 79.1 N
   D. 84.3 N

8. Sid has an isoceles triangle wedge with a perimeter of 36 cm and an IMA = 1.2. Find the width of his wedge.
   A. 10 cm
   B. 15 cm
   C. 24 cm
   D. 30 cm

9. Samarth has a pulley with AMA = 3.5 that pulls a load of weight W up 36 m in 60 s. If the equivalent power of the pulley is 0.67 hp, find his pulling force F_p on the pulley.
   A. 229 N
   B. 238 N
   C. 241 N
   D. 256 N

10. Which one of these is **not** a property of self-threading screws?
    A. Tight thread pitch
    B. Presence of a drill point
    C. Loose grip on material
    D. Steel-based composition

11. Roly-poly Rebecca rolls down a frictionless inclined plane, accelerating from rest to 28 m/s in 5 s. Find the IMA of the plane.
    A. 1.40
    B. 1.75
    C. 2.45
    D. 2.80

12. Which one of these simple machines **is not** utilized in a can opener mechanism shown above?
    A. Class I Lever
    B. Gear
    C. Wedge
    D. Class II Lever

Questions 13–16 refer to Lucky Luke in a baseball game. A pitcher throws a ball with $v = 43$ m/s horizontal to the ground towards Luke. Assume there is negligible drop in the pitch. Luke's bat is 90 cm long and he holds his left hand 70 cm from the tip. His bicep is roughly 70 cm long from elbow to hand with a muscle load around 5 cm from the elbow joint. The mass of a standard baseball is roughly 0.145 kg.

13. What simple machines are used in the system?
    A. Two Class II Levers
    B. Two Class III Levers
    C. Class II Lever + Class III Lever
    D. Class I Lever + Class III Lever

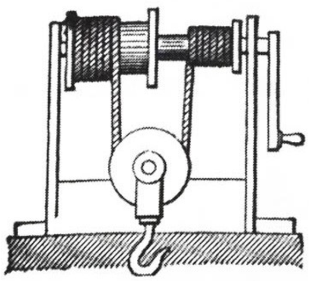
14. Find the IMA of the entire bat-arm system.
    A. 0.02
    B. 0.07
    C. 14.0
    D. 49.0

15. Luke hits the ball and it has exit velocity $v = 40$ m/s. The baseball is at a height $h = 0.5$ m upon contact and it travels 100 m before hitting the ground. Find the launch angle of this ball if it is under 45°.
    A. 11°
    B. 19°
    C. 26°
    D. 30°

16. Computing the input force needed using the IMA gets you get a very large value. Why?
    A. Mass of the bat is non-negligble
    B. Short effort arm induces high error
    C. Air resistance is non-negligble
    D. IMA is linear and does not consider rotational motion

Thirsty Thomas needs your help in working his city's water well! He is using a differential windlass with a handle length of 28 cm and IMA = 11.2.

Use this information to answer questions 17–20 about this apparatus.



17. Which one of the following is a possible combination of the diameters of the small and large axles, d_s and d_l?

    A. 12 cm, 17 cm C. 6 cm, 16 cm
    B. 5 cm, 25 cm D. 2 cm, 3 cm

18. The cylindrical bucket of water at the bottom of the well, when empty, is 1 kg and has an interior radius of 27 cm. Thomas exerts 60 N of force to lift it up at a constant velocity. What is the height of the water in the bucket if the density of water is $\rho = 1\,\text{g/cm}^3$?

    A. 14.5 cm C. 29.5 cm
    B. 28.9 cm D. 29.9 cm

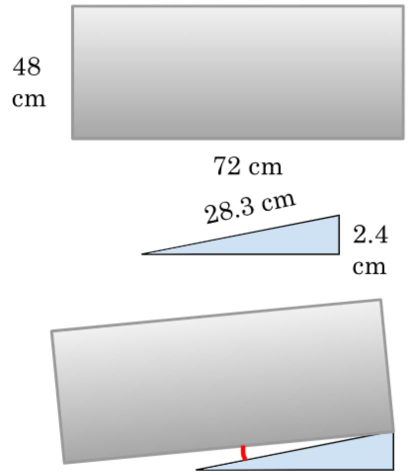
19. Which one of the following adjustments would increase the IMA?

    A. Decreasing the small axle radius.
    B. Increasing the handle length and decreasing the large axle radius.
    C. Reducing both axle radii
    D. Lubricating all axles

20. Thomas's arm gets sore, so he brings a robot that rotates the handle at angular acceleration α. The vertical displacement of the **empty** bucket after t seconds, in meters, is $\Delta y = \alpha t^2/n$ where n is a positive integer. Find n.

    A. 20 C. 56
    B. 40 D. 80

Questions 21–23 refer to Mason the Mason and his wedge system below, where Mason lifts up a heavy slab of platinum using a triangular wedge.



21. Find the IMA of the wedge.

    A. 5.875
    B. 5.895
    C. 11.75
    D. 11.79

22. The wedge just touches the slab after Mason pushes the wedge, with the other corner being a fixed pivot. Given that the minimum force needed to keep the slab up is $F_{\text{min}} = 617\,\text{N}$ and the density of the material is $\rho = 21.45\,\text{g/cm}^3$, find the width of the slab.

    A. 18.93 cm
    B. 19.12 cm
    C. 19.97 cm
    D. 20.84 cm

23. Find the red angle sketched in the diagram.

    A. 1.78°
    B. 2.01°
    C. 2.35°
    D. 2.96°

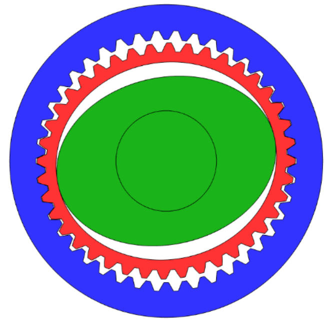
24. Aidan has a set of 12 gears with the following number of teeth: 8, 12, 14, 16, 17, 18, 24, 27, 32, 35, 36, 40. He can arrange these gears to get an IMA = 1.5 in exactly $a \cdot b!$ ways where b is maximized. Find $a \cdot b$.
   - A. 30
   - B. 35
   - C. 50
   - D. 60

25. Arthur has a three-gear system with gears A, B, C has tooth ratio 24 : 20 : 30, respectively. At t_0, gear C accelerates radially at α. At some time $t_1 > t_0$, gear B has made 30 revolutions, and 5 seconds later, Gear A has made 50 revolutions. Find α.
   - A. $1.439\,\text{rad/s}^2$
   - B. $1.725\,\text{rad/s}^2$
   - C. $1.840\,\text{rad/s}^2$
   - D. $1.913\,\text{rad/s}^2$

26. Jimmy hosts a "tug-of-torque" war, using a rod of length 2026 m with a fulcrum on one end. 2026 people apply equal forces of 1 N from distances 1 m, 2 m, etc. from the fulcrum. The closest force is clockwise, and each adjacent force alternates direction. Determine the direction and magnitude of the overall torque.
   - A. 1014 N m, clockwise
   - B. 1014 N m, counterclockwise
   - C. 1013 N m, counterclockwise
   - D. 1013 N m, clockwise
   - E. There is no net torque

27. Yeming links n pulleys of AMA = a in series. Interestingly, the total mechanical advantage of this system is the same as the combined mechanical advantage of each pulley alone. Find a in terms of n.
   - A. $a = \sqrt{n}$
   - B. $a = 1/(n^n)$
   - C. $a = \sqrt[n]{n}$
   - D. $a = 1/n$
   - E. $a = \sqrt[n-1]{n}$

28. Izzy is doing a project on harmonic/strain wave gears. She needs a section on their real-life applications. What are some of them? **Select all that apply.**



   - A. Robotic arms
   - B. Engine pistons
   - C. Agricultural machinery
   - D. Conveyor belt mechanics
   - E. Bicycle gear systems

29. **Select all of the following** that are examples of compound machines.
   - A. Steering wheel
   - B. Hammer (placing nails)
   - C. Scissors
   - D. Drill bit
   - E. Hand drill

30. **Select all of the following** that represent the IMA of a specific or general simple machine.
   - A. $F_{\text{in}}/F_{\text{out}}$
   - B. $d_{\text{in}}/d_{\text{out}}$
   - C. $d_{\text{out}}/d_{\text{in}}$
   - D. Rise over run for inclined planes
   - E. Angular velocity ratio of final to initial gears in a gear train

31. Sourish has four pulleys of IMA 2, 3, 4, and 5, respectively. He needs to make a compound pulley of integer IMA with these four rules:

    1. He can connect any two pulleys in series and connect previously formed compound pulleys.
    2. While not conventional, in this problem, he may combine two or more pulleys to form a new one with the IMA being the sum of all the individual IMA values.
    3. He must use *at least* **two** of the four pulleys to meet the definition of a compound machine.
    4. He **cannot reuse** any machine.

    Find the range containing the lowest IMA ≥ 5 that cannot be created.

    A. 20-29
    B. 30-39
    C. 40-49
    D. 50-59
    E. 60+

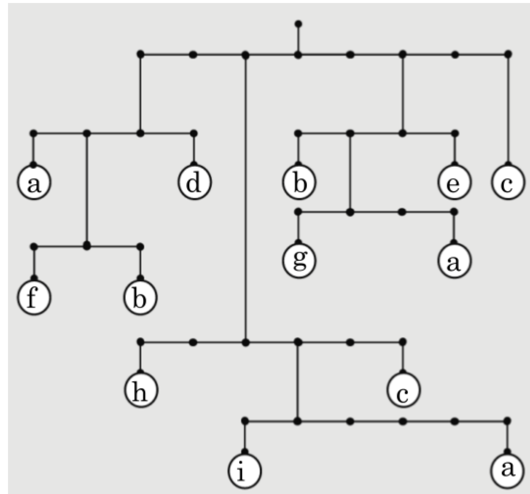
32. In a lever competition, Krish is tasked with balancing a 103.4 N force over at least 5.6 cm using a maximal input force of 27.8 N. If his lever may be no longer than 49.2 cm in total, and the material he is using makes it only 72 % efficient, **select all possible** AMAs of the lever.

    A. 3.20
    B. 3.78
    C. 5.49
    D. 5.72
    E. 6.01

33. Alexis the apprentice has six congruent wedges linked entirely in series. She needs to cut a log with force of 471 N to break. However, due to a recent hand injury, she cannot exert more than a 34 N force. Additionally, each wedge cannot resist more than a 400 N force. **Select all possible** AMAs of the wedges.

    A. 1.67
    B. 1.71
    C. 1.84
    D. 1.87
    E. 1.94

This question refers to the MOAB[1] below. Assume all rods and strings are massless and all line segments are equal. All masses are integers.



34. You are given that $3a+4b=c$ and $2a+6b=22$. Find the total mass of the system.

    A. 101
    B. 105
    C. 113
    D. 117
    E. 121

35. The angle of repose for a specific type of granite is 39°. Suhas pours a mixture of this granite on an infinitely wide inclined plane with angle 7°, forming a conical pile. It has a height of 5 cm (measured from tip of cone to surface perpendicular to plane) and an elliptical base with minor to major axis ratio of 0.981. Find the volume of this cone.

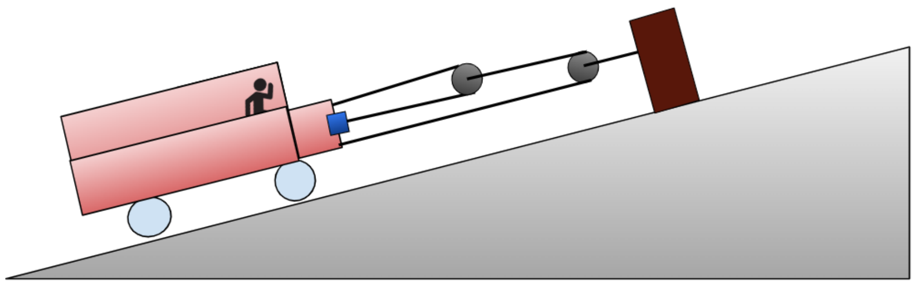
    A. 192 cm^3
    B. 196 cm^3
    C. 205 cm^3
    D. 211 cm^3
    E. 220 cm^3

[1] Mother of all Balances. Unfortunately, not the Mother of all Bloons for my BTD enjoyers.

Section B: Free Response

There are 5 multi-part questions in this section, for a total of 80 points.

1. **Aneesh's Automobile Adventure.** Aneesh's 2200 kg truck gets stuck in the cornfields of Indiana as he travels up a 12° degree incline. He uses an electric truck winch motor with nylon cables (originating from the middle cable segment on the truck at the blue box) and links this system to an upright log in the distance. Both the cable and pulley blocks (the black circles) have negligible mass, and the total mass of the truck includes the mass of Aneesh.



   (a) [2 pts] Determine the IMA of the winch system.

   (b) [3 pts] The cable is well-worn and frayed, so the maximum total tension of any cable segment is 2000 N. Find the minimum static friction coefficient μ_s such that the winch can support the truck without snapping the cable.

   (c) [3 pts] Fortunately, Aneesh has some supplies to reinforce the cable and now it can withstand anything. He powers the winch and accelerates up the incline. As he passes through a muddy patch with a kinetic friction coefficient $\mu_k = 0.18$ at 1 m/s, the truck accelerates at $0.25\,\text{m/s}^2$. Compute the tension of the cable (in N) originating from the winch at that instant.

   (d) [2 pts] If the IMA in part (a) is increased, would the instantaneous power consumption of the winch motor in part (c) be greater, lesser, or the same? Justify.

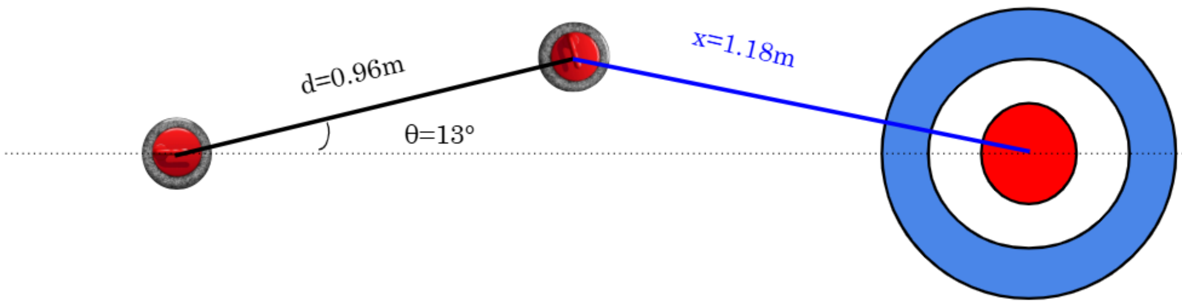
2. **LeBron's Levers.** Our glorious King James needs your help to make an impossible shot! He attempts to launch a basketball across the entire court using machines. A class one lever of length 4 m and mass 1 kg is placed so that the fulcrum of height 50 cm is a distance 30 m away from the hoop. A ball of mass 600 g is placed on the far side of the lever (going further away from the target hoop), and a mass 2 kg is suspended a distance h_2 m above the other end of the lever. You may assume that the hoop is exactly 3 meters tall, and that both masses are point masses located exactly at the ends of the lever.

   (a) [3 pts] To achieve a launch angle 47°, what is the required IMA of the lever? You may assume that the ball will leave the lever at an angle perpendicular to the plane at its final point of motion.

   (b) [3 pts] Once the ball is launched, what is the required initial velocity v_0 (in m/s) in order for it to reach the hoop? You may assume that the ball does not accelerate after leaving the lever.

   (c) [4 pts] What is the required energy E_1 (in J) that must be applied to the near side of the lever to achieve this final velocity? Note that from the parallel axis theorem on any rigid body we have that $I = I_{\text{cm}} + md^2$, where $I_{\text{cm}} = 1/12 \cdot mL^2$ is the moment of inertia for the rod about the center of mass, m is the mass of the object, d is the perpendicular distance between the new axis and the axis through the center of mass.

3. **Shuffled Scuffboard.** Consider the mini-curling setup below.



This is a portion of a much larger, smooth curling board surface. The target has three regions; the blue and white strips (worth 2 and 4 points, respectively) have thickness 8 cm and the 'bullseye' red circle (worth 7 points) has radius 6 cm. Each mini-curling puck has radius 2.8 cm and mass 45 g. A puck must be fully in a region for points to be scored; if it is in two regions, the lower score is taken. Assume all collisions are perfectly elastic.

(a) Answer some questions about the mechanics of this setup:

i. [1 pt] To launch the puck, you use your bicep to push it. What class lever are you using?

ii. [2 pts] Your bicep has IMA = 0.11. The curling stone has a handle of length 4.5 cm and a resistive band of length 4 cm. What is the total IMA of the arm-band system?

(b) [3 pts] You attempt to hit the second puck, which is 13° north relative to the horizontal and 0.96 m away. You want to find the range of angle(s) such that you still hit the puck. Find the minimum such angle (relative to the horizontal).

(c) [3 pts] You hit the puck such that the resultant velocity of the second puck is strictly horizontal. At what angle must you launch the first puck such that this happens?

(d) [4 pts] You decide to try sliding the first puck itself straight for the target. If the table has $\mu_k = \mu_s = \mu = 0.05$, find the minimum force (in N), given contact time of $t = 1.5$ s, from the bicep such that it lands in the white, scoring four points.

(e) You decide to hit the second puck instead.

i. [1 pt] How many degrees under the horizontal must the second puck travel to be dead center?

ii. [3 pts] Find the initial angle of the first puck (relative to the horizontal) such that the second puck can land dead center.

iii. [3 pts] At the angle you found in (ii), find the maximum velocity (in m/s) of the first puck such that the second puck is dead center.

4. **Kid's Menu.** You know how restaurants have kid's menus with puzzles to keep them distracted?

Part	A	B	C	D	E	F
(a)	1	0	1	?	∞	∞
(b)	1	?	0	0	0	0
(c)	∞	?	?	1	?	?
(d)	?	1	1	∞	?	1

Use the diagram on the next page and the table above to answer the following questions. Assume all pulleys and strings are massless and frictionless and grey bars to be immovable. Any pulley directly connected to an immovable bar is translationally fixed. From left to right, the coaxial pulleys have diameter ratios of 2 : 4 : 5, 1 : 3 : 5, and 1 : 2. Assume the small angle effect on the topmost coaxial pulley is negligible. The value in the corresponding cell in the table is the mass of the box in units of beavers: 1 bvr = 1/9.81 kg. Note that a mass of infinity (∞) indicates the box is immovable and a question mark indicates the mass of the box is to be determined. All masses are initially at rest and are released simultaneously.

(a) [2 pts] What is the mass of box D, in beavers, if the system is in equilibrium?

(b) [2 pts] What is the mass of box B, in beavers, if the system is in equilibrium?

(c) [3 pts] Identify all possible sets of masses (M_B, M_C, M_E, M_F) such that the system is in equilibrium.

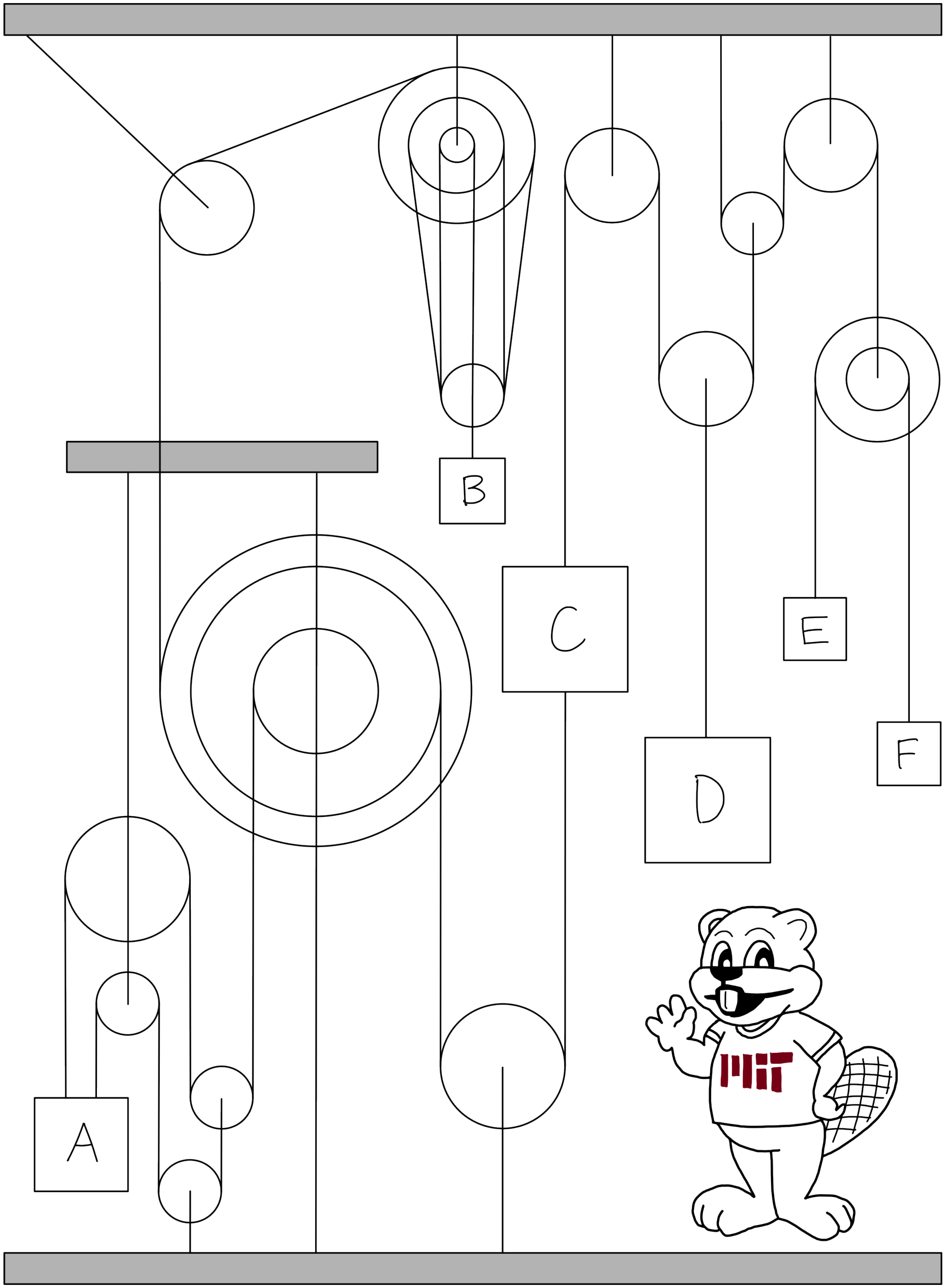
(d) *Warning: These questions may be tedious...*

i. [5 pts] Suppose $M_A = M_E = M > 0$. There is a range of masses $M_{\text{min}} < M < M_{\text{max}}$ (in beavers) such that box C accelerates upwards. $M_{\text{min/max}}$ can be expressed in the form $(a \pm \sqrt{b})/c$ where a, b, c are relatively prime positive integers. Find them.

ii. [5 pts] Let $M = 1$. Compute the rates and directions of acceleration of box A and E, in m/s^2.

iii. [3 pts] After one second, what is the rate at which the potential energy of the whole system is being converted into kinetic, in W?

(e) [0 pts] Name the fellow in the corner.


A
B
C
D
E
F
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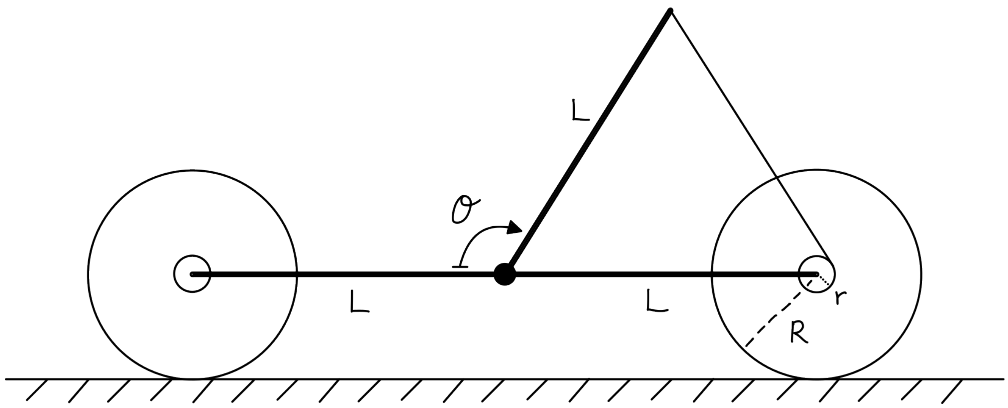
5. **Mousetrap Vehicle.** Consider the diagram of a mousetrap vehicle below.



The vehicle is constructed from a rigid uniform plate (the chassis) of length $2L$ and mass M, four massless wheels of equal radius R connected in pairs by two rigid massless axles of equal radius r, and a rigid massless lever of length L connected to the midpoint of the chassis by a torsion spring (the mousetrap) with a spring constant κ on one end and tied to a massless string on the other end that is spooled around the axle on the right such that it does not slip. The torsion spring on the lever imparts a linear-elastic torque in the counterclockwise direction; more formally: $\tau = -\kappa\theta$, where θ is the angle the mousetrap is opened to. The static and kinetic coefficients of friction between the floor and the wheels are $\mu_s = \mu_k = \mu$. All other interactions are frictionless.

(a) [1 pt] What direction does the vehicle move? Right or left?

Express your answers to parts (b) through (d) in terms of the given variables and g.

(b) [2 pts] When the vehicle is at rest, what is the normal force on each wheel?

(c) [2 pts] What is the maximum horizontal acceleration of the vehicle without the wheels slipping?

(d) [4 pts] Derive the expression for the torque on the right axle as function of θ.

Compute your answers to parts (e) and (f) in the required units using the parameters: $L = 20\,\text{cm}$, $M = 700\,\text{g}$, $R = 7\,\text{cm}$, $r = 5\,\text{mm}$, $\kappa = 6\,\text{N cm/rad}$, and $\mu = 0.7$. Note that the diagram is not to scale.

(e) [3 pts] Evaluate the maximum angle the mousetrap can be opened without the wheels slipping, in degrees.

(f) [6 pts] Calculate the distance traveled if the mousetrap closes from 100° to 15°, in meters.

(g) [2 pts] Does the distance in part (f) change if each wheel has mass (e.g., 10 g)? Explain why or why not using qualitative arguments.